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TWO-STAGE STOCHASTIC LINEAR PROGRAMMING WITH RECOURSE: A CHARACTERIZATION OF LOCAL REGIONS USING RESPONSE SURFACE METHODOLOGY

THESIS

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Table of Contents

List of Tables

List of Figures

List of Appendices

Abstract

The LP recourse problem applies to two-stage optimization problems where uncertainty in resource availability of the second stage hinders informed decision making. The recourse function affords a way to compensate "later" for an error in prediction "now." The literature provides a rich body of work on the optimization of such problems, but little research has been accomplished regarding the characterization of the surface in the local region of optimality, in particular sensitivity analysis. A decision maker faced with considerations other than the modeled objective function must be presented with a way to estimate the impact of operating at non-optimal decision variable values. This work develops and demonstrates a technique for characterizing the surface using response surface methodology. Specifically, the flexibility and utility of RSM techniques applied to this class of problems is demonstrated, and a methodology for characterizing the surface in the local region using a low-order polynomial is developed.

1.0: Introduction

This research extends the work of Lt. Col. T. Glenn Bailey's Doctoral Dissertation at the University of Texas, Austin (Bailey 1995). Bailey utilized response surface methodology (RSM) in characterizing the surface of several two-stage stochastic linear programming with recourse (2SSLP) problems as a post-optimization analysis process. The problems he examined included APL1P, CEP-1, PGP2, 4-Term, and 20-Term. With the exception of 20-Term, the problems were small (15 decision variables or fewer), and the RSM analysis was predicated with knowledge gained during the optimization process.

The goals of this work were twofold: First, to develop a structured methodology for the RSM-based analysis of this class of problems independent of any optimization knowledge, and second, to demonstrate the methodology on "large" problems (60 or more decision variables). The sponsor for the research was Lt. Col. Bailey, and the computer resources used were the property of the Air Force Institute of Technology.

This work develops and demonstrates a structured methodology for designing and analyzing a low-order polynomial approximation for the 2SSLP family of problems. These problems are often difficult to analyze due to high dimensionality and a highly variant stochastic nature. A polynomial metamodel is useful for providing a decision maker with sensitivity analysis, and provides an easily computed estimate for changes in decision variable values.

Chapter 2 introduces the problem with a rigorous mathematical definition, and provides a historical background. Chapter 3 develops the methodology for the work, and

Chapter 4 provides computational results for two problems considered. Chapter 5 provides an overview of results, and directions for further research.

2.0: Two-Stage Stochastic Linear Programming with Recourse: A Characterization of Local Regions using Response Surface Methodology

This thesis develops and implements a technique for characterizing the response surface of a two-stage stochastic linear programming (LP) with recourse problem using response surface methodology (RSM). Specifically, we demonstrate the flexibility and utility of RSM techniques applied to this class of problems, and develop a methodology for characterizing the surface in a local region using a low-order polynomial.

The LP recourse problem applies to two-stage optimization problems where uncertainty in resource availability of the second stage hinders informed decision making. The recourse function affords a way to compensate "later" for an error in prediction "now" (Kall and Wallace, 1994). The general form of the problem is

$$\begin{aligned} Z(x) &= \operatorname*{Min}_{x}[cx + E[h(x,\omega,\mathbf{T})]] \\ &\ni x \in X \end{aligned} \tag{1}$$

where

$$\begin{aligned} h(x,\omega,T) &= \operatorname*{Min}_{y} dy \\ &\ni \mathbf{W}y = \omega + \mathbf{T}x, y \geq 0 \end{aligned} \tag{2}$$

The second-stage RHS vector ω and matrix $\mathbf{T}$ can contain both stochastic and deterministic elements, and $\mathbf{W}$ is the matrix of second-stage technological coefficients (Bailey, Jensen and Morton 1996). The objective is to find x^*, where $Z(x^*) = \min Z(x)$.

Dantzig first proposed the recourse problem as a way to extend linear programming beyond deterministic here-and-now decision problems, and generalized it to an *m*-stage problem (Dantzig 1955). Dantzig notes that only the first stage activities are determined, the later stage variables are random and dependent on the first stage variables, and that for any feasible choice of first-stage variables the following stages are

feasible. This last property is relatively complete recourse (Kall and Wallace 1994), an attribute we assume for two-stage problems. Wets (1966) proved that the feasible probability space of the problem is both convex and continuous, thus guaranteeing a unique optimal solution.

The literature provides significant research in optimization of problems in this class. Ermoliev (1983, 1988) explored gradient and quasi-gradient optimization methods, and introduced the use of the dual variables for gradient information. Higle and Sen (1991) employed decomposition methods for optimization, and Sen, Doverspike and Cosares (1994) used decomposition methods and validated the optimization with a simulation of the system studied. Bailey (1995) used the projective gradient among other nonlinear optimization techniques, and studied estimator variance reduction techniques. Morton and Wood (1997) demonstrated techniques for bounding the optimal solution of the problem.

RSM integrates statistical experiment design, regression techniques, and elementary optimization methods to identify and fit an appropriate response surface which characterizes a system that is generally not fully understood. RSM seeks to identify the input variables which affect an output (or response) variable, and fit an appropriate response surface. The response surface used is generally a first- or second-order polynomial fitted using the least-squares approach (see Myers and Montgomery 1995, Box and Draper 1987).

Bailey (1995) utilized elements of RSM on the recourse problem by showing that designed experiments allow a systematic low-order polynomial characterization of the surface. He included sampling techniques to reduce the variance of the estimator of

$E[h(x, \omega, \mathbf{T})]$, since excessive estimator variance often creates an indeterminate approximation of the known piecewise-convex surface. Bailey found the greatest variance reduction using Latin hypercube sampling (LHS), a method we employ in this paper (for the theoretical background of LHS, see Mckay *et al* 1979, and Mckay 1988).

3.0: *Design Methodology*

Our approach begins with the assumption that x* is known, then continues with providing an appropriate experimental design to characterize the response surface in the optimal region. We empirically demonstrate the utility of this technique using the sequential design strategy depicted in Figure 3-1.

1) Screen decision variables
   a) Design and fit Resolution IV design augmented with axial points
   b) Fit model of linear and pure quadratic terms
   c) Retain factors with predetermined significance level
2) Choose experimental design for final model
   a) Central Composite Design
   b) Minimal Bias design
   c) Simplex Lattice if equality constraints are present
3) Choose factor ranges and design center
   a) Design centering strategy is based on analysis objectives
   b) Factor ranges based on analysis objectives and *a priori* knowledge of problem characteristics
4) Sample problem for experimental design, and fit model
5) Model validation
   a) Statistical validation
      i) Residual analysis
      ii) ANOVA significance
      iii) Lack of fit
   b) Sampling validation
      i) Randomly sample design space
      ii) Compare predicted values to actual observations
6) Canonical analysis
   a) Analyze curvature of fitted model
   b) Compare to known characteristics of problem

Figure 3-1: General Design Methodology.

3.1: CCD and Minimal Bias Design

Two sources of error in an empirical model are variance and bias, the latter referred to as *systematic errors* (Myers and Montgomery 1995). Bailey (1995) addresses several sampling methods to reduce model variance, but left model bias for later work. We construct and use a minimal bias design by adapting standard minimal-bias design techniques to the specific nature of the recourse problem. Bailey (1995) demonstrates a

Central Composite Design (CCD) to highlight the effectiveness of LHS in reducing the variance of the estimator of E[h(x, ω, **T**)]. The CCD has the advantage of simple implementation while still providing a large amount of information about the surface, but will not protect against model bias.

For initial screening purposes, a small composite design is constructed using a resolution IV foldover of a Plackett-Burman augmented with 2 axial points for each parameter. The Plackett-Burman portion is used to estimate the largest possible number of terms in the fewest runs. This design screens only linear and pure quadratic effects, and reflects the assumption that a significant parameter will have either a significant linear or quadratic term. Second-order interactive terms are not considered in accordance with this assumption. Terms higher than second order are not addressed, an assumption we make under the sparsity of effects principle (Myers and Montgomery 1995). Furthermore, as the feasible space of the recourse problem is convex our use of second-order terms is appropriate.

The choice of range for each decision variable is motivated by several factors. First, it must be large enough to provide the decision maker a practical and meaningful choice. Second, the range must be feasible, e.g. a nonnegative decision variable with a center value of five cannot have a range of six. Third, empirical studies have shown a relatively high variance in the region of optimality (see Bailey 1995); therefore the range must induce a sufficiently large change in the response to differentiate the effect of a variable from the effect of random variance (both system variance and estimator variance). Finally and conversely, the range must be sufficiently small to allow an accurate second-order polynomial approximation of the surface in the local region of

optimality, the region of interest. For each problem, the same factor range is used for each factor in every design to allow for unbiased comparisons, e.g if one factor has a range of ± 5 all factors have a range of ± 5.

Several options exist for modeling screened factors in the reduced factor design. First, these variables can be held to a coded value of zero. Second, we can group these variables, either randomly, or by an external criteria, and vary them as a group across the design. Third, we can allow each screened variable to independently vary across the design using an assigned distribution. The first and third alternatives provide insight into the accuracy of the individual screening, while the second option provides knowledge on the group effects.

After screening, we determine the center of the design space, construct the design for the first-stage variables, and fit the polynomial approximation of the problem. The resulting metamodel can then predict and provide sensitivity analysis within the design space (see Box and Draper 1987, Neter *et al.* 1996).

We adopt Box and Draper's (1987) minimum bias design, which is near-exact when used to minimize the bias of a second-order model of a cubic surface. Our design is adapted from a resolution V central composite design. We achieve minimum bias by scaling the design parameters such that the design moments, M_{ij}, equal the region moments, μ_{ij}, (a single subscripted moment denotes a pure moment, a double subscript is an interactive moment). For the second-order design, with the origin defined as the design center, all region moments through order 5 are zero, with the exception of the following:

$$\mu_{2_i} = \int_O w(x) x_i^2 dx$$

$$\mu_{4_i} = \int_O w(x) x_i^4 dx$$

$$\mu_{22_{ij}} = \int_O w(x) x_i^2 x_j^2 dx$$

where

$$w(x) = \frac{1}{\int_O dx}$$

(Box and Draper 1987, Myers and Montgomery 1995).

Over a coded cuboidal design space the region of operability O is defined by the [-1, 1] range of each coded variable. The weight function *w(x)* is

$$w(x) = \frac{1}{\int_{-1}^{1}\int_{-1}^{1}\cdots\int_{-1}^{1} dx} = \frac{1}{2^i}$$

which is the inverse of the volume of the *i*-dimensional hypercube with side length 2. The limits of integration are constant and therefore interchangeable, resulting in the following values for the region moments (complete computations are shown in Appendix A)

$$\mu_2 = \frac{1}{3}$$

$$\mu_4 = \frac{1}{5}$$

$$\mu_{22} = \frac{1}{9}.$$

The relevant design moments are represented as

$$M_{2_i} = \frac{1}{N}\sum_{n=1}^{N} x_i^2$$

$$M_{4_i} = \frac{1}{N}\sum_{n=1}^{N} x_i^4$$

$$M_{22_{ij}} = \frac{1}{N}\sum_{n=1}^{N} x_i^2 x_j^2$$

where N = number of total design points. Equating the design moments and region moments results in:

$$n_f a^2 + 2\alpha^2 a^2 = N/3$$

$$n_f a^4 + 2\alpha^4 a^4 = N/5$$

$$n_f a^4 = N/9$$

$$n_c + n_f + 2k = N$$

where n_f = the number of factorial design points, a = halflength of one edge of the minimal bias hypercube, k = number of axial points, α = multiplier for the axial points, and n_c = number of center points (Box and Draper 1987). The design parameters are not always integer, but the general form of the design is robust to inexactness in the parameters (Myers and Montgomery 1995). The use of any balanced design for the $\pm$ sign matrix guarantees that the odd moments through order 5 are zero. Axial points contribute to the estimation of the pure quadratic terms, and multiple center points facilitate estimation of pure error and lack of model fit.

The center of the design space must be chosen to provide the richest sensitivity analysis possible. Therefore, we center the design in such a way that the extremes of the cuboidal region have a roughly equal Z(x) value, exploiting this "flatness" for our sensitivity analysis. We do not move more than 0.6 coded units away from the optimal

for any parameter, and only move parameters with statistical significance in order to retain the validity of the screening design. While a rectangular region (not restricted to cuboidal) could encompass an even “flatter” region, the coded parameter estimates would be more difficult to interpret. In all cases only uncoded parameter estimates are provided.

We demonstrate three methods of centering the design. Our first method uses the gradient obtained at the optimal solution by projecting from the optimal solution out. The gradient is calculated using the method shown by Ermoliev (1983, 1988) where for the i^{th} realization of ω and **T** the negative gradient is

$$-\nabla_{Z_{ik}} = c + \pi_{ik}\mathbf{T}$$

and for x_k the unbiased estimate of the negative gradient for N samples is

$$-\nabla_{Z_k} = -\frac{1}{N}\sum_{i=1}^{N}\nabla_{Z_{ik}}.$$

Within the region of interest the upper and lower limits of the design variables are based on the gradient. This method requires sampling at only one point.

Our second method of locating the design center uses the linear and quadratic estimates from the screening design. The polynomial approximation from the axial screening design is

$$f(x) = \beta_0 + \sum_i \beta_i x_i + \sum_i \alpha_i x_i^2$$

where

$$\frac{\partial}{\partial x}(f(x)) = 0$$

for

$$x_k = \frac{-\beta_k}{2\alpha_k},\ k = 1,2,\ldots,i$$

thus yielding a center point for the design.

The last centering methodology centers the design at the optimal solution. The first two strategies reflect an interest in a location least susceptible to variation in the parameters. The third strategy reflects an interest in characterizing the response to variation in the parameters in the immediate vicinity of the optimal solution.

3.2: Modified Simplex-Lattice Koshal

Two of the three problems we examine in this work contain equality constraints. The Simplex-Lattice (SL) design is applicable when equality constraints in the problem preclude a standard CCD (see Myers and Montgomery 1995). The CCD requires that each variable be capable of independent movement; however, an equality constraint requires any increase in one variable be appropriately offset by decreasing one or more associated variables. For the generalized equality constraint

$$\sum_{i=1}^{q} x_i = 1$$

a *{q, m}* simplex-lattice, for modeling q parameters in a polynomial of degree m, is formed by equally spacing $m+1$ points on a simplex such that

$$x_i = 0, \frac{1}{m}, \frac{2}{m}, \ldots, 1, \qquad i = 1, 2, \ldots, q$$

and then forming all ordered sets $(x_1, x_2, \ldots, x_q)$ which satisfy the equality constraint (Myers and Montgomery, 1995). The simplex vertices

$$(1,0,\ldots,0),\ (0,1,\ldots,0),\ \ldots,(0,0,\ldots,1)$$

estimate pure linear effects, and the remaining points estimate interactions. In general, a set with k nonzero elements estimates a k^{th}*-order* interaction. For $m=2$, the fitted polynomial is

$$f(x) = \sum_{i=1}^{q} \beta_i x_i + \sum_{i<j}^{q} \sum \beta_{ij} x_i x_j$$

Estimating a constant term in the SL requires the *a priori* removal of one term to allow estimation of the intercept.

The simplex-lattice models the response for the entire range of the equality constraint, and we assume the response for the range of equality constraints in this class of problems to be too complex to model with acceptable accuracy using a single polynomial. As with the CCD and the minimal-bias model, we seek to model only the region about the optimal solution. We restrict the model to the region of interest using a set of deviation variables d_i such that

$$\sum_{i=1}^{q} d_i = 0$$

where d_i denotes the deviation of x_i from the optimal solution. In this manner the design is restricted to the local region. Since no set $(c,0,\ldots,0), c \neq 0$ can satisfy this constraint, no direct estimation of linear terms can be made. Additionally, any deviation variable equals the negative of the sum of the remaining variables, forcing a singularity in the regression matrix. We therefore augment the design with axial points at the extreme feasible values for each variable. Since estimating pure quadratic terms, or an intercept term, results in a singular regression matrix, the general model follows the standard SL form. The Koshal form of estimating interaction terms is used to limit the number of samples that must be made (See Myers and Montgomery 1995).

3.3: Validation:

We evaluate the validity of our polynomial approximation in the following manner. We first apply the standard measures of adequacy (residual analysis, lack of fit, etc.) using the Shapiro-Wilk test (Neter *et al.* 1996) for normality, and the Levene test (Neter *et al.* 1996) for nonconstant variance. We then sample additional points within the design space, and compare the predicted values of the polynomial metamodel to the values provided by the original model of (1).

4.0: Problem Descriptions and Computational Results

4.1: 20-Term Problem Description and Computational Results

The first problem we address is 20-Term, a vehicle allocation problem between a central depot and 20 outlying terminals (hence the name 20-Term). 20-Term is ideal for our study as it is a large problem (63 decision variables and 40 random variables) and the optimum is known (Bailey 1995). The decision variables x_1 through x_{21} are trailers (600 available), while x_{22} through x_{42} are tractors (400 available); together, these comprise the existing fleet, at a modeled cost of zero. A complete "vehicle" requires one tractor and one or two trailers. Variables x_{43} through x_{63} are rental units (one combined tractor-trailer) with a daily cost of 100, and constrained to a maximum of 10,000 rental units. These constraints are represented mathematically as

$$\sum_{i=1}^{21} x_i = 600$$

$$\sum_{i=22}^{42} x_i = 400$$

$$\sum_{i=43}^{63} x_i \leq 10{,}000$$

and represent the first stage of the problem (corresponding to (1)). The **T** matrix allocates the decision variables among the central depot and outlying terminals without any stochastic representation, and is expressed mathematically as

$$-x_j - x_{j+42} = 0, j = 1,2,\ldots,21$$

$$-x_j - x_{j+21} = 0, j = 22,23,\ldots,42$$

The two equality constraints lack the necessary degrees of freedom to construct a Central Composite Design (CCD). Consequently, x_1 and x_{22} are used as dependent variables such that

$$x_1 = 600 - \sum_{i=2}^{21} x_i$$

$$x_{22} = 400 - \sum_{i=23}^{42} x_i .$$

Removing x_1 and x_{22} from consideration projects the modeled surface onto the surface representing the outlying terminals. The forty independent random variates represent supply and demand at each of the outlying terminals; the lack of supply and demand variables for the central depot support the use of the central location as a staging point for the outlying terminals.

At the optimal solution there are thirty-eight nonzero parameters (Bailey 1995), including x_1 and x_{22}. All parameters with a zero value are *a priori* removed to allow movement in both directions, and a parameter range of five is used with 200 replications at each point to estimate the value. The parameter screening reduces the number of variables under consideration to thirteen (screening design results are in Appendix D).

The optimal value of x_{25} is too small to allow a spherical region CCD, so the space is modeled as a cuboidal region (face centered CCD). A 2_{V}^{13-5} design base gives 256 factorial runs, 26 axial points, and 42 center points for a total of N=324 runs. The relatively large number of center replications stabilizes estimator variance and allows good estimation of lack of fit. We demonstrate all three centering strategies with the CCD. Summary statistics for the models are shown in Table 4-1.

Table 4-1: 20-Term CCD Summary Statistics

Centering Strategy	Optimal	Quadratic	Gradient
Rsquare	0.997	0.991	0.997
Rsquare Adj	0.996	0.990	0.996
RMSE	399.147	224.154	363.333
ANOVA Significance	<.001	<.001	<.001
Lack of Fit	<.001	0.601	<.001
Residual Normality	0.770	0.299	0.893
Residual Variance	0.062	0.422	0.094
Max Response	281570	262869	280543
Min Response	254657	254684	254778
Std Dev(Response)	6230.107	2220.225	6028.360

The CCD summary statistics suggest a good approximation, with normally distributed, constant variance residuals. The optimal centered model indicates no lack-of-fit, and model significance is strong. The quadratic-centered model has a high lack-of-fit; the extremes of this design are well outside the range of the initial screening design, and screened variables could possibly be significant in the new region. However, good ANOVA model significance coupled with good residual analysis allows the analysis to proceed. The RMSE and the standard deviation of the response indicate the quadratic centering does produce the desired effect – a near-optimal location least susceptible to change in the parameters. The gradient centering provides the best all-around model, with the highest R^2 and a 10% reduction in RMSE over the optimal centered model, as well as good residual analysis and no indication of lack of fit.

A canonical analysis of the optimal-centered 13-variable CCD provides sensitivity analysis for the fitted model (see Myers and Montgomery 1995, Box and Draper 1987). The cuboidal design is not rotatable, therefore the standard errors for the eigenvalues of the second-order model cannot be extracted directly from the regression. Instead, we apply the Bisgaard and Ankenman (1998) approach of rotating the decision variable values using the matrix of eigenvectors associated with the original matrix of

second-order terms, and fitting the model using the rotated data. In this model, the eigenvalues are the coefficients of the pure quadratic terms, and the standard error of the coefficients are used as the standard error of the eigenvalues. Table 4-2 contains the results of the canonical analysis, and Table 4-3 contains the 95% confidence intervals for the eigenvalues. The eigenvector corresponding to the largest eigenvalue represents the maxima ridge for the system, while the eigenvector corresponding to the smallest eigenvalue represents the minima ridge. There are no zero eigenvalues, indicating that there is no pure ridge system.

Table 4-2: 20-Term Maximal and Minimal Ridge Eigenvectors

Eigenvalue	**62.449**	**7.875**
X4	0.514	-0.003
X11	0.217	0.042
X23	0.012	0.000
X24	-0.059	-0.002
X25	-0.597	0.000
X31	-0.192	-0.059
X32	-0.315	0.722
X33	-0.317	-0.686
X34	-0.225	0.039
X36	-0.026	-0.021
X39	0.185	0.000
X43	-0.082	0.012
X62	0.034	0.005

Table 4-3: Eigenvalues of 20-Term CCD

Eigenvalue	**Std Error**	**Lower 95%**	**Upper 95%**
62.449	1.054	60.382	64.515
61.669	1.046	59.619	63.718
54.470	1.910	50.727	58.213
47.203	0.975	45.292	49.113
40.490	1.597	37.359	43.621
37.399	1.011	35.418	39.380
36.266	1.050	34.209	38.323
35.490	3.995	27.660	43.319
34.460	1.078	32.348	36.572
26.075	2.854	20.480	31.670
23.501	1.322	20.909	26.092
18.094	2.944	12.323	23.865
7.875	1.282	5.362	10.388

Figure 4-1 is a graph of the observed response along the minimal and maximal eigenvectors. The region of interest is dominated by second-order terms, and Figure 4-1 demonstrates the accuracy of the second-order ridge analysis, with the maximal ridge observations dominating the minimal ridge observations at each point.

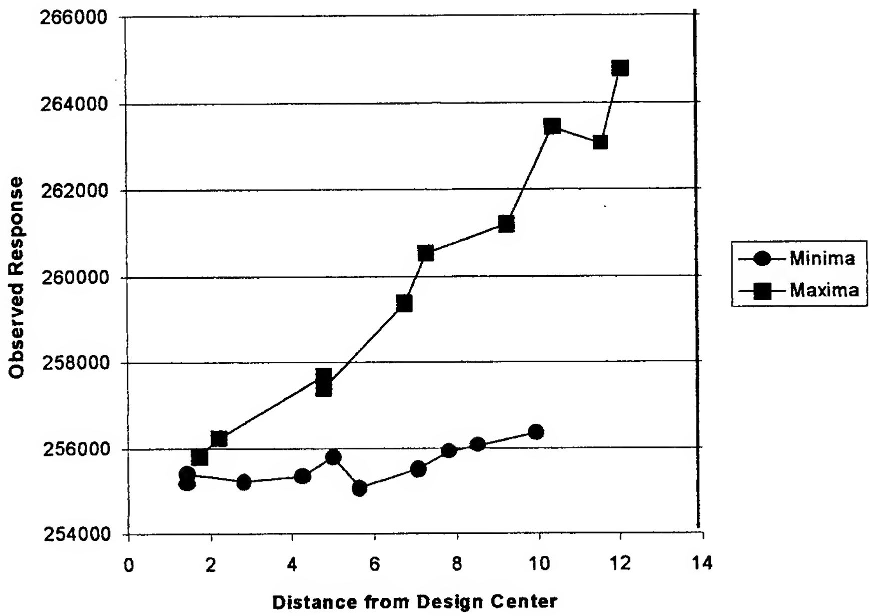

Figure 4-1: Observations along Maxima and Minima Ridge Eigenvectors (20-Term)

The minimal bias model is constructed in much the same manner as the CCD. For the 256 factorial point CCD base, the design moment equations are satisfied with $a=0.6$, $\alpha=3$, and 20 center runs, centered at the optimal solution. Our range of interest is ± 5 units for each variable; unfortunately, the optimal value of $x_{25} = 6$ is too small to accommodate the range of movement required for the axial points. Consequently, x_{25} is dropped from the minimal bias portion of experimentation. Table 4-4 lists the summary

statistics for the regression, and Figure 4-2 is a graph of the predicted response and the observed response.

Table 4-4: 20-Term Minimal Bias Summary Statistics

Centering Strategy	Optimal
Rsquare	0.946
Rsquare Adj	0.939
RMSE	503.167
ANOVA Significance	<.0001
Lack of Fit	<.0001
Residual Normality	0
Residual Variance	<.0001
Max Response	264468.8
Min Response	254762.1
Std Dev(Response)	2036.162

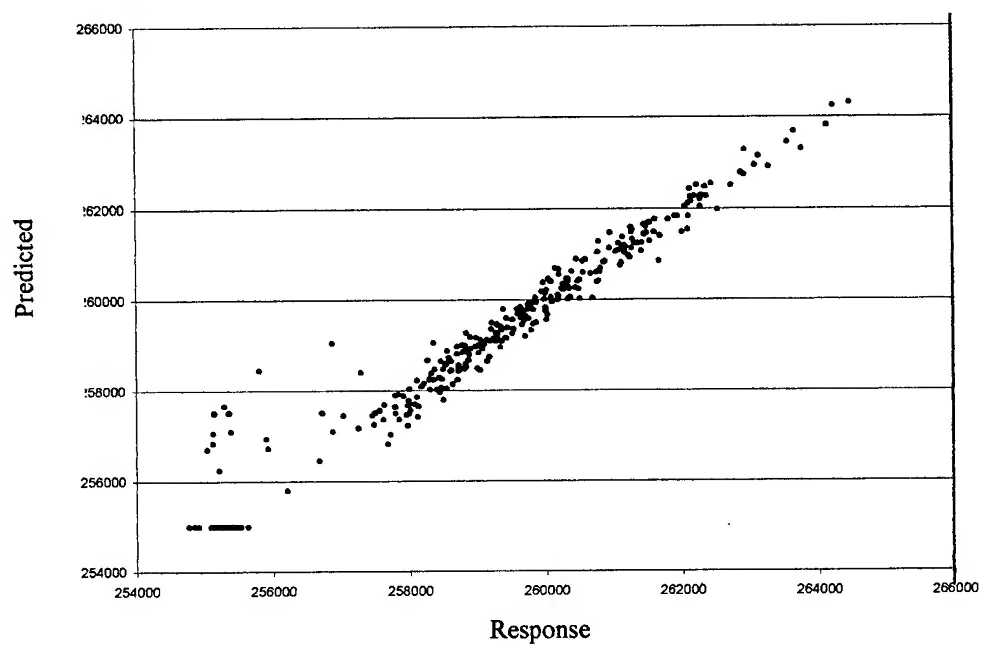

Figure 4-2: Minimal Bias Response by Predicted Response

The minimal bias model has a slightly worse R^2 than the CCD, and a root mean squared error (RMSE) 20% higher than the CCD. The deletion of x_{25} appears to have had a noticeable effect on the regression. The residuals also violate the model requirement of normal distribution. This is seen in Figure 4-2, where the predicted values and the

observed values in the lower portion of the graph are not on the line, indicating a region of poor prediction and high residual values. If there exists a set of factor levels which produce poor estimation and a peak in residual values, this could possibly be remedied by regressing using weighted least-squares instead of the standard least-squares procedure used (Neter et al. 1996). We instead recommend the use of the CCD as it allows the use of all significant variables.

20-Term has two equality constraints and one inequality constraint, each with mutually exclusive parameters. This allows blocking the problem into three parameter groupings. We apply the modified SL to the first constraint, holding x_{22} through x_{63} at their optimal solution values and modeling all interactions within the x_1 through x_{21} block. As with the CCD, a range of ± 5 for each nonzero variable is modeled, with all zero valued parameters excluded. The remaining sixteen parameters are modeled with 240 two-way interactive samples (two for each interaction), 16 axial points, 32 6-way interactive points to allow for estimation of error, and 12 center points to allow an estimate of lack of fit. Summary statistics are in Table 4-5, and the deviation matrix is in appendix P. Figure 4-3 is error plotted against response for the non-axial portion of the design.

Table 4-5: 20-Term MSLK Summary Statistics

Centering Strategy	Optimal
Rsquare	0.976
RSquare Adj	0.974
RMSE	17417.16
ANOVA Significance	<.0001
Lack of Fit	<.0001
Residual Normality	<.0001

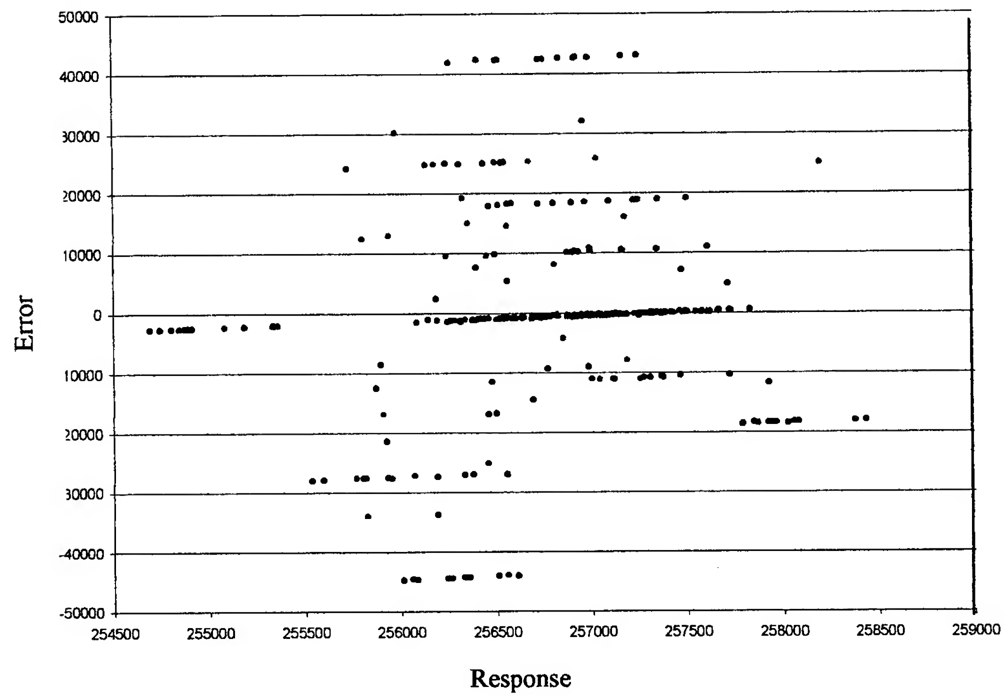

Figure 4-3: 20-Term MSLK Error by Response

Results for this design are not promising. RMSE is high, the residual analysis shows a lack of normality, and nonconstant variance is visually apparent in Figure 4-3. The axial points are not in the local region, and appear to reduce the effectiveness of the model in the region of optimality. A possible remedy would be to regress using weighted least-squares, minimizing the impact of the axial points. No further analysis of this design will be pursued.

The CCD family of designs produces consistently good models. For the optimal-centered design, the CCD is the obvious choice, and to provide a location least sensitive to parameter changes, we recommend using either the gradient-centered method, or the quadratic-centering method with a screening design at the design center to choose variables which will improve lack of fit.

As a final validation step, we generate 5,000 random points within the design space, and allow factors with an optimal value of zero to vary uniformly from zero to two, while all non-zero variables range uniformly ± 5 from their optimal value. Predicted values are computed and the percentage errors plotted against the distance from the design center in Figure 4-4. The worst error was less than 8%, with an average error of 2.5%.

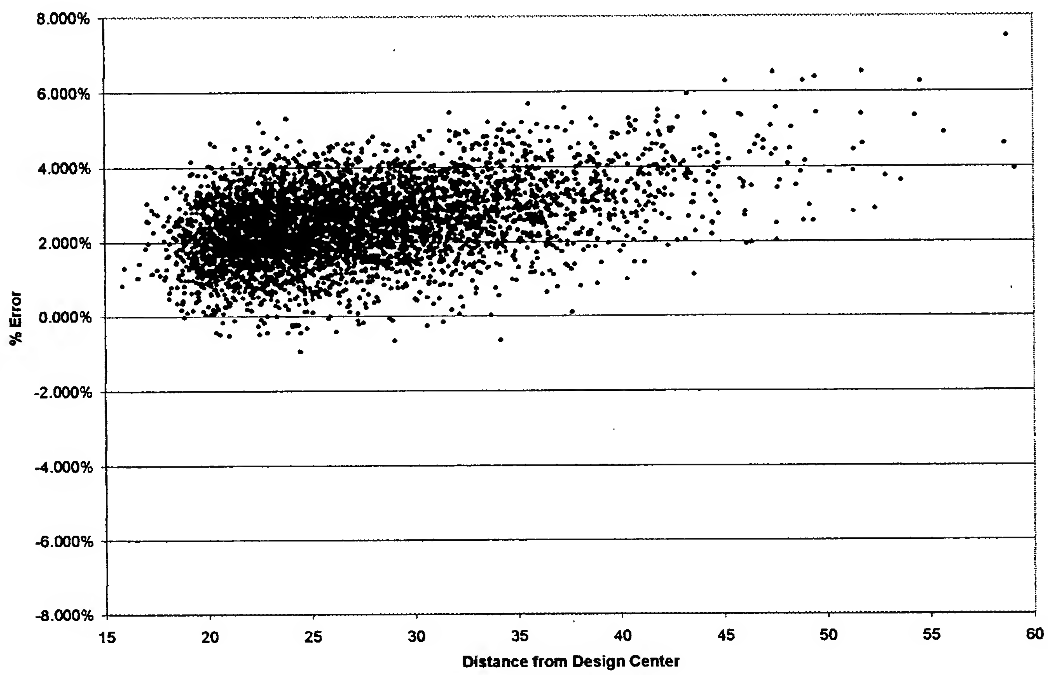

Figure 4-4: 20-Term Validation Error by Distance (Uncoded Space)

4.2: SSN Problem Descriptions and Computational Results

The next problem analyzed is SSN, a multi-commodity network flow model with 31 nodes, 89 links (decision variables), 86 demand pairs (stochastic), and 706 routes. A route is comprised of no more than 3 links. Each link may be assigned a capacity, with no more than 1,008 total units of capacity. This constraint is represented mathematically as

$$\sum_{i=1}^{89} x_i \leq 1{,}008$$

and is the only decision variable constraint. The literature provides neither a description of the acronym SSN nor units for the objective function; neither are required for the analysis.

At the optimal solution the constraint is active, a situation referred to as a *binding multi-component constraint* (Myers and Montgomery 1995). One possible design choice for this type of region is a computer-generated design (Myers and Montgomery 1995). We instead relax the constraint and center a CCD at the optimal solution, and impose the constraint on the final model.

Ten replications of 400 runs each at the optimal solution provides an estimate of variance (approximately 1.9), and an estimate of the optimal solution (approximately 10.04). The high variance prompts the use of 800 replications at each point. An initial screening design using a cuboidal region with decision variable range size of five (all factors with a value less than five are *a priori* removed) provided poor results – an indefinite fit (indicated by the presence of both positive and negative eigenvalues) in a known convex space (Wets 1966). The summary statistics for this design are in table 4-6.

Table 4-6: Summary Statistics for SSN Cuboidal Screening Design

Centering Strategy	Optimal
RSquare	0.872204
RSquare Adj	0.760947
RMSE	0.942029
Max Eigenvalue	1.194027
Min Eigenvalue	-0.97663

The range is not large enough to differentiate between random variation and variation induced by decision variable changes, and indicates the need for a larger decision variable range. The use of a cuboidal range must be abandoned and each factor range is determined as

$$r_i = \min(10, x_{i,op})$$

where r_i is the factor range for the *ith* variable and $x_{i,op}$ is the optimal solution value for the *ith* variable. All variables with a value of zero are not considered.

At the optimal solution there are fifty-five nonzero parameters (Morton 1999). The factor screening indicates that including thirteen variables results in an acceptably high R^2 of 0.87, while the use of all fifty-five results in only a small increase in R^2 to 0.95. Forty-one variables are statistically significant in the region of interest, and a complete model of the region would require all forty-one; computational considerations for this work restrict the analysis to fewer factors. Consequently thirteen factors are chosen using a stepwise screening procedure (summary statistics and parameter estimates for the design are in Appendix J). A 2_{V}^{13-5} design base is used, with 256 factorial runs, 26 axial points, and 18 center points for a total of N=300 runs. As with 20-Term, the relatively large number of center replications stabilizes estimator variance and allows good estimation of lack of fit. The alternate centering strategies push the design center out of the feasible region and are not used in this problem.

The CCD summary statistics in Table 4-7 suggest a good approximation, with normally distributed, constant variance residuals. There is evidence to suggest lack of fit due to the high variance of the system, but the R^2 of 0.896, good ANOVA model significance, and excellent residual analysis allows the model analysis to proceed.

Table 4-7: SSN CCD Summary Statistics

Centering Strategy	Optimal
Rsquare	0.896
Rsquare Adj	0.884
RMSE	0.886
ANOVA Significance	<.0001
Lack of Fit	0.389
Residual Normality	0.514
Residual Variance	0.560

As with 20-Term, canonical analysis of the CCD provides sensitivity analysis for the fitted model. The design is not rotatable, and the Bisgaard and Ankenman (1998) method of estimating the standard errors of the eigenvalues is used here as well. Table 4-8 contains the results of the canonical analysis, and Table 4-9 contains the 95% confidence intervals for the eigenvalues. For brevity only the maximal and minimal ridge eigenvectors are included. Eight of the thirteen eigenvalues cannot be distinguished from zero at the 95 % confidence level, indicating pure second-order ridge systems within the design space. The eigenvalues are all near zero, indicating very low curvature within the design region, and sampling along the minimal and maximal ridge eigenvectors confirms this. Figure 4-5 is a plot of the observations along the maximal and minimal eigenvectors; the observations along the distinct eigenvectors yield indistinct results, affirming that a first-order analysis is sufficient for the region.

Table 4-8: SSN Maximal and Minimal Ridge Eigenvectors

Eigenvalue	0.011	-0.002
X12	-0.477	0.002
X17	0.058	0.036
X19	0.000	0.523
X21	0.047	0.000
X24	-0.014	0.006
X30	-0.002	-0.702
X33	0.393	-0.018
X40	0.326	0.005
X41	-0.112	-0.005
X43	-0.226	0.008
X53	-0.012	-0.482
X60	0.038	-0.005
X64	-0.664	0.003

Table 4-9: Eigenvalues of SSN CCD

Eigenvalue	Std Error	Lower 95%	Upper 95%
0.011	0.002	0.008	0.014
0.008	0.002	0.005	0.011
0.003	0.001	0.002	0.005
0.003	0.001	0.001	0.005
0.003	0.001	0.001	0.004
0.001	0.002	-0.002	0.005
0.001	0.001	-0.001	0.002
0.001	0.001	-0.001	0.003
0.001	0.002	-0.003	0.004
0.000	0.001	-0.001	0.002
0.000	0.002	-0.004	0.003
0.000	0.001	-0.002	0.002
-0.002	0.001	-0.003	0.000

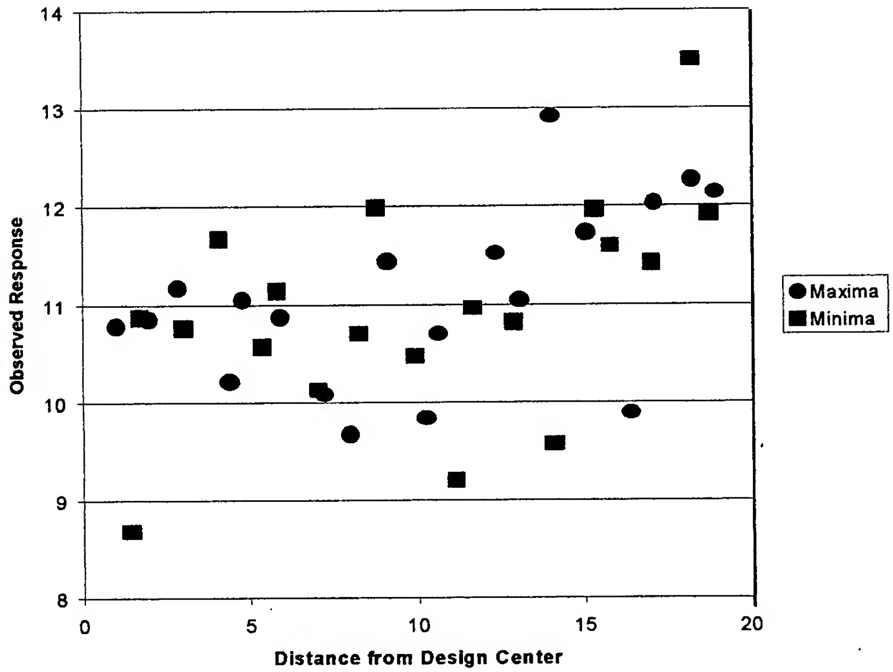

Figure 4-5: Observations along Maxima and Minima Ridge Eigenvectors (SSN)

The first-order direction of greatest change, maxima, is represented by the vector of linear estimates (Myers and Montgomery 1995); any orthogonal vector is the direction of least change, minima, (for **L** the vector of linear estimates, **P** an alternative direction vector, **LP**=0 is the smallest magnitude of change possible; therefore **P** orthogonal to **L** is a direction of least change). Table 4-10 contains the maxima vector and a minima vector (other minima vectors exist; any vector orthogonal to the maxima is a minima vector), and Figure 4-6 is a plot of the observations along the maxima and minima vectors.

Table 4-10: Maxima and Minima First-Order Vectors for SSN CCD

Variable	Maxima	Minima
X12	0.000	0.000
X17	-0.355	0.500
X19	-0.106	-0.500
X21	-0.539	-0.500
X24	-0.078	0.500
X30	-0.059	0.500
X33	-0.196	0.500
X40	-0.267	-0.500
X41	-0.106	0.500
X43	-0.131	0.500
X53	-0.036	-0.500
X60	-0.140	0.500
X64	-0.114	-0.500

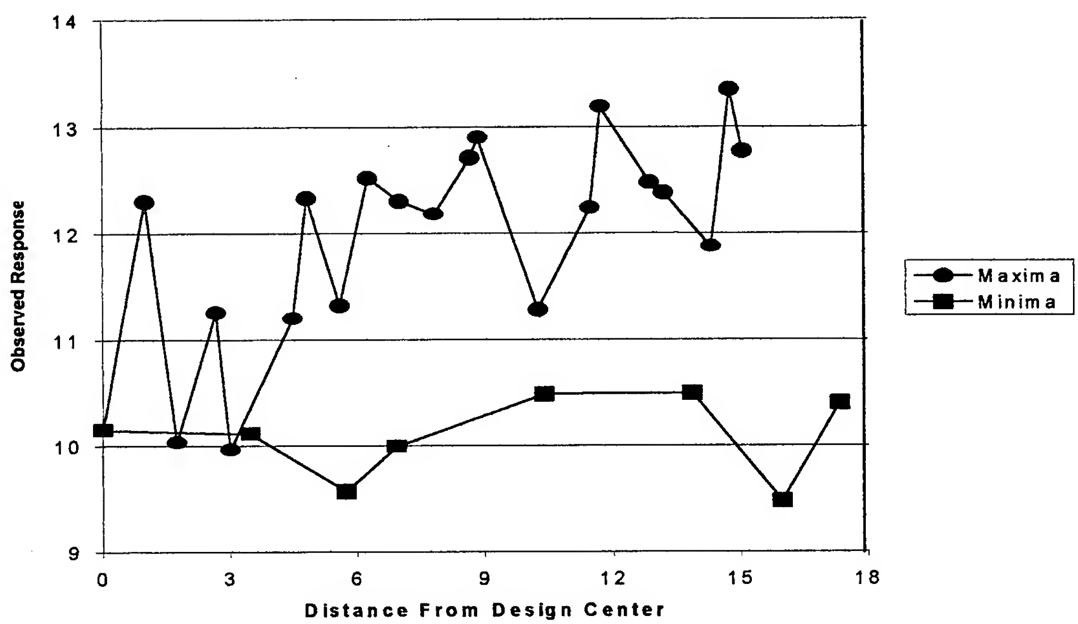

Figure 4-6: Observations along First-Order Maxima and Minima (SSN)

Figure 4-6 shows the accuracy of the direction vectors obtained from the CCD, and also demonstrates the highly variant nature of SSN. There is virtually no change in the minima direction, while the observations along the maxima show a clear rising tendency.

The use of only a small subset of the statistically significant parameters for the CCD preclude the type of whole-model random prediction validation used for 20-Term.

4.3: Empirical Conclusions

The empirical models of 20-Term and SSN provide an easily computable estimate for a decision maker. The methodology presented in Chapter 2 is validated by the accuracy of the models, in particular the sensitivity analysis.

5.0: Contributions and Future Research

This research extends the methodology for characterizing local regions of the two-stage stochastic linear programming with recourse family of problems, using empirical modeling techniques. We provide a framework to accurately characterize the surface with a low-order polynomial and provide sensitivity analysis in the optimal region. We also demonstrate the utility of response surface methodology for this class of problems, and show the flexibility of RSM through the use of several different techniques to adapt to specific problems.

While no alphabet-optimal designs were used in this work, the literature suggests the G and Q-optimal family of designs may prove useful in the characterization of this class of problems, since these designs specifically seek to stabilize prediction variance within the design region. The binding multi-component constraint in SSN could be modeled using a computer-generated design to produce a more accurate characterization of the optimal region than the CCD. Our approach of relaxing the binding constraint is not always a realistic possibility, and, in the case of a binding, non-releasable constraint, a computer-generated design could provide the necessary structure to characterize the surface. The nature of the specific problems studied led to the use of cuboidal design regions. A spherical design region is generally more desirable, and for problems where this is feasible, will most likely lead to more accurate prediction within the design region. In particular, a spherical design would allow the use of a minimum-bias design with all significant variables. An exploration of the minimum-bias design on a problem with the necessary freedom of range is valuable in minimizing error within the design region.

High variance hindered the analysis of SSN. Total variance is the sum of estimator variance and system variance, and although nothing can be done to minimize system variance, the total variance can be minimized through the use of estimator variance reduction techniques. While Latin hypercube sampling is used throughout this paper for this purpose, other possible techniques include antithetic variates and control variates. Furthermore, LHS requires independent variables, which are assumed to be present for each problem studied; however, for correlated variables LHS may not prove suitable. A comparative study of several different variance reduction techniques under different correlation assumptions should be undertaken on high-variance problems as SSN.

Finally. statistical significance of variables was a key factor in screening factors within the design range. The relatively few number of significant variables in 20-Term led to the highly accurate characterization of the surface with a small polynomial, but with SSN the number of variables had to be restricted due to the large number of variables that were statistically significant. The use of a small-composite design could possibly result in a higher-accuracy design by allowing the efficient modeling of more factors with a manageable number of runs.

APPENDIX A: Terminology.

We use the following concepts and terminology throughout this paper.

Coded Variable: A variable transformed by scaling the minimum and maximum to –1 and 1, respectively (see Myers and Montgomery 1995, Box and Draper 1987). See *Uncoded Variable*.

Cuboidal region: A special case of the rectangular region where the range of each parameter is equal. See *Rectangular Region*.

Deterministic Model: A model whose inputs are known with certainty (Banks *et al.* 1996). Contrast with *Stochastic* model.

Extreme Point: Given a function $f(x)$ and a point x* in the domain of $f(x)$, any ε>0, and an s, if ∀ ε>0 and all s such that $\|s\| \le \varepsilon$, either:

1. $f(x^* + s) \le f(x^*)$

2. $f(x^* + s) \ge f(x^*)$

Then x* is an extreme point of f. In case (1) above, x* is a local maximum, and in case (2) x* is a local minimum (Schmidt and Davis 1981).

Global Maximum: Given a set of all local maximum points (x_1, x_2, x_3,…, x_i) for a function f, the global maximum x_h defined to be $x_h = \max[x_1, x_2, x_3,\ldots, x_i]$ (Schmidt and Davis 1981). See *Extreme Point*.

Global Minimum: Given a set of all local minimum points (x_1, x_2, x_3,…, x_i) for a function f, the global minimum x_h defined to be $x_h = \min[x_1, x_2, x_3,\ldots, x_i]$ (Schmidt and Davis 1981). See *Extreme Point*.

Karush-Kuhn-Tucker (KKT) conditions: A set of conditions necessary for a point to be an optimum (Bazaraa *et al.* 1990). KKT conditions are both necessary and sufficient for constrained optimization problems (Schmidt and Davis 1981).

Random Number Generator: An algorithm for providing pseudo-random numbers. This paper uses a *Linear Congruential Generator*, where $Z_i = (aZ_{i-1} + C) \text{modulo } m$ (Law and Kelton, 1991).

Rectangular Region: A region where the range of each parameter is independent of all other parameters. See *Cuboidal Region*.

Rotatable Design: A design in which the variance of the estimator is a function only of the distance from the design center (Box and Draper 1987).

Stationary Point: See *Extreme Point*. An alternate definition is a point x* where the gradient of the function is zero (Winston 1994).

Stochastic model: A model where at least one input is a random variable (Banks *et al.* 12). Contrast with *Deterministic* model.

Uncoded Variable: A variable represented in original units. See *Coded Variable*.

APPENDIX B: Moment Computations.

$$\mu_2 = \int_O w(x)x_i^2 dx = \int_{-1}^{1}\int_{-1}^{1}\cdots\left\{\int_{-1}^{1}\frac{1}{2^i}x_k^2 dx_k\right\}dx_1\ldots dx_{k-1}dx_{k+1}\ldots dx_i = \frac{1}{2^i}2^{i-1}\frac{2}{3} = \frac{1}{3}$$

$$\mu_4 = \int_O w(x)x_i^4 dx = \int_{-1}^{1}\int_{-1}^{1}\cdots\left\{\int_{-1}^{1}\frac{1}{2^i}x_k^4 dx_k\right\}dx_1\ldots dx_{k-1}dx_{k+1}\ldots dx_i = \frac{1}{2^i}2^{i-1}\frac{2}{5} = \frac{1}{5}$$

$$\mu_{22} = \int_O w(x)x_k^2 x_l^2 dx = \int_{-1}^{1}\int_{-1}^{1}\cdots\left\{\int_{-1}^{1}\left\{\int_{-1}^{1}\frac{1}{2^i}x_k^2 dx_k\right\}x_l^2 dx_l\right\}dx_1\ldots dx_{k-1}dx_{k+1}\ldots dx_{l-1}dx_{l+1}\ldots dx_i$$

$$= \frac{1}{2^i}\int_{-1}^{1}\int_{-1}^{1}\cdots\left\{\int_{-1}^{1}\frac{2}{3}x_l^2 dx_l\right\}dx_1\ldots dx_{l-1}dx_{l+1}\ldots dx_i = \frac{1}{2^i}\frac{2}{3}2^{i-2}\frac{2}{3} = \frac{1}{9}$$

APPENDIX C: Parabolic Design Centering Computations.

$$f(x) = \beta_0 + \sum_i \beta_i x_i + \sum_i \alpha_i x_i^2$$

so

$$\frac{\partial}{\partial x_i}(f(x)) = \beta_i + 2\alpha_i x_i$$

$$\frac{\partial}{\partial x_i}(f(x)) = \beta_i + 2\alpha_i x_i = 0$$ (setting partial derivative with respect to x_i to zero)

$$\beta_i + 2\alpha_i x_i = 0$$

$$2\alpha_i x_i = -\beta_i$$

$$x_i = \frac{-\beta_i}{2\alpha_i}, \alpha_i \neq 0$$

APPENDIX D: 20-Term Screening Design Results.

Response: Response

Summary of Fit

RSquare	0.912907
RSquare Adj	0.900926
Root Mean Square Error	2052.249
Mean of Response	275662.7
Observations (or Sum Wgts)	216

Lack of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack of Fit	98	762895824	7784651	21.3888
Pure Error	91	33120257	363959	Prob>F
Total Error	189	796016081		<.0001

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	271170.32	174.2928	1555.8	0.0000
4	-623.3938	237.4169	-2.63	0.0094
4*4	1867.2617	1312.017	1.42	0.1563
11	-423.4396	237.4169	-1.78	0.0761
11*11	1592.502	1312.017	1.21	0.2263
23	-1043.801	237.4169	-4.40	<.0001
23*23	1562.801	1312.017	1.19	0.2351
24	-547.6445	237.4169	-2.31	0.0222
24*24	1056.07	1312.017	0.80	0.4219
25	-1249.936	237.4169	-5.26	<.0001
25*25	1962.9422	1312.017	1.50	0.1363
31	-425.3867	237.4169	-1.79	0.0748
31*31	1132.5585	1312.017	0.86	0.3891
32	-660.2916	237.4169	-2.78	0.0060
32*32	879.01807	1312.017	0.67	0.5037
33	-764.1467	237.4169	-3.22	0.0015
33*33	787.20664	1312.017	0.60	0.5492
34	-611.2801	237.4169	-2.57	0.0108
34*34	1423.7077	1312.017	1.09	0.2792
36	-855.3613	237.4169	-3.60	0.0004
36*36	1427.8315	1312.017	1.09	0.2779
39	-509.739	237.4169	-2.15	0.0331
39*39	1255.0798	1312.017	0.96	0.3400
43	646.30283	237.4169	2.72	0.0071
43*43	284.916	1312.017	0.22	0.8283
62	425.7261	237.4169	1.79	0.0745
62*62	-2245.31	1312.017	-1.71	0.0887

APPENDIX E: 20-Term CCD Design Results.

Response: Response

Summary of Fit

RSquare	0.996531
RSquare Adj	0.995895
Root Mean Square Error	399.1465
Mean of Response	264883
Observations (or Sum Wgts)	324

Lack of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack of Fit	232	41139976	177327	3.0888
Pure Error	41	2353825	57410	Prob>F
Total Error	273	43493801		<.0001

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	50	1.24935e10	2.4987e8	1568.374
Error	273	43493801.2	159318	Prob>F
C Total	323	1.2537e+10		<.0001

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	1503610.2	1591824	0.94	0.3457
X11*X23	2.6107684	0.997866	2.62	0.0094
X11*X24	3.37054	0.997866	3.38	0.0008
X23*X24	25.832694	0.997866	25.89	<.0001
X24*X25	7.0478868	0.997866	7.06	<.0001
X11*X31	-2.779321	0.997866	-2.79	0.0057
X11*X32	-9.124528	0.997866	-9.14	<.0001
X31*X32	18.751966	0.997866	18.79	<.0001
X11*X33	-6.008188	0.997866	-6.02	<.0001
X31*X33	16.026274	0.997866	16.06	<.0001
X32*X33	41.729022	0.997866	41.82	<.0001
X11*X34	-1.975712	0.997866	-1.98	0.0487
X31*X34	4.223032	0.997866	4.23	<.0001
X32*X34	11.815232	0.997866	11.84	<.0001
X33*X34	15.597613	0.997866	15.63	<.0001
X34*X36	2.4440648	0.997866	2.45	0.0149
X25*X39	-14.03803	0.997866	-14.07	<.0001
X11*X43	-6.974411	0.997866	-6.99	<.0001
X4*X11	8.8046915	0.997866	8.82	<.0001
X4*X23	2.9062381	0.997866	2.91	0.0039
X4*X25	-15.72449	0.997866	-15.76	<.0001
X4	990.65336	489.5964	2.02	0.0440
X4*X4	64.08403	10.84804	5.91	<.0001
X11	144.56282	832.3194	0.17	0.8622
X11*X11	38.240406	10.84804	3.53	0.0005
X23	-2377.273	285.937	-8.31	<.0001
X23*X23	44.455534	10.84804	4.10	<.0001
X24	-2160.368	737.1017	-2.93	0.0037
X24*X24	36.71196	10.84804	3.38	0.0008
X25	-888.8892	135.2441	-6.57	<.0001
X25*X25	68.839778	10.84804	6.35	<.0001
X31	-1482.89	307.4997	-4.82	<.0001
X31*X31	22.03542	10.84804	2.03	0.0432
X32	-2014.012	393.7514	-5.11	<.0001
X32*X32	19.13702	10.84804	1.76	0.0788

X33	-2206.076	436.3253	-5.06	<.0001
X33*X33	21.069174	10.84804	1.94	0.0531
X34	-2459.035	440.3417	-5.58	<.0001
X34*X34	43.18785	10.84804	3.98	<.0001
X36	-2015.673	413.1288	-4.88	<.0001
X36*X36	44.784666	10.84804	4.13	<.0001
X39	-1548.626	368.9154	-4.20	<.0001
X39*X39	42.191168	10.84804	3.89	0.0001
X62	6933.9054	4386.128	1.58	0.1151
X62*X62	31.328842	10.84804	2.89	0.0042
X4*X43	-8.20122	0.997866	-8.22	<.0001
X43	-6144.206	8316.241	-0.74	0.4607
X43*X43	9.3969059	10.84806	0.87	0.3871
X43*X62	-22.96189	11.33358	-2.03	0.0437
X32*X36	1.7881153	0.997866	1.79	0.0743
X24*X43	-1.879555	0.997866	-1.88	0.0607

APPENDIX F: 20-Term Minimal Bias Design Results.

Response: Response
Summary of Fit

RSquare	0.946082
RSquare Adj	0.938934
Root Mean Square Error	503.1673
Mean of Response	259413.9
Observations (or Sum Wgts)	300

Lack of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack of Fit	245	65842343	268744	5.1243
Pure Error	19	996462	52445	Prob>F
Total Error	264	66838806		<.0001
Max RSq				
0.9992				

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	3028128.5	638197.3	4.74	<.0001
X4*X11	10.782175	3.494217	3.09	0.0022
X23*X24	20.892113	3.494217	5.98	<.0001
X11*X32	-12.25127	3.494217	-3.51	0.0005
X31*X32	13.247014	3.494217	3.79	0.0002
X31*X33	9.4100612	3.494217	2.69	0.0075
X32*X33	31.499883	3.494217	9.01	<.0001
X32*X34	12.065418	3.494217	3.45	0.0006
X33*X34	10.777376	3.494217	3.08	0.0023
X4*X43	-7.401196	3.494217	-2.12	0.0351
X11*X43	-8.71441	3.494217	-2.49	0.0132
X34*X43	-10.41168	3.494217	-2.98	0.0032
X4	754.70084	1349.064	0.56	0.5763
X4*X4	58.513029	4.337515	13.49	<.0001
X11	224.19558	1372.754	0.16	0.8704
X11*X11	46.270288	4.337515	10.67	<.0001
X23	-2397.806	151.9513	-15.78	<.0001
X23*X23	59.700475	4.337515	13.76	<.0001
X24	-2797.826	255.8447	-10.94	<.0001
X24*X24	41.13873	4.337515	9.48	<.0001
X31	-1760.195	153.924	-11.44	<.0001
X31*X31	42.366866	4.337515	9.77	<.0001
X32	-2141.127	225.3012	-9.50	<.0001
X32*X32	37.587771	4.337515	8.67	<.0001
X33	-2691.638	203.5631	-13.22	<.0001
X33*X33	40.46728	4.337515	9.33	<.0001
X34	1087.8992	1352.794	0.80	0.4220
X34*X34	56.07097	4.337515	12.93	<.0001
X36	-2218.545	165.1367	-13.43	<.0001
X36*X36	53.186959	4.337515	12.26	<.0001
X39	-2089.885	147.8232	-14.14	<.0001
X39*X39	57.564489	4.337515	13.27	<.0001
X43	-13557.83	3325.77	-4.08	<.0001
X43*X43	18.458021	4.337515	4.26	<.0001
X62	-1634.208	251.7799	-6.49	<.0001
X62*X62	27.616426	4.337515	6.37	<.0001

APPENDIX G: 20-Term Gradient Centered CCD Results.

Response: Response

Summary of Fit

RSquare	0.99693
RSquare Adj	0.996367
Root Mean Square Error	363.3332
Mean of Response	264537.2
Observations (or Sum Wgts)	324

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	50	1.17021e10	2.3404e8	1772.905
Error	273	36038998.5	132011	Prob>F
C Total	323	1.17382e10		0.0000

Lack of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack of Fit	232	34194797	147391	3.2768
Pure Error	41	1844201	44981	Prob>F
Total Error	273	36038998		<.0001
Max RSq				
0.9998				

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	1336418.5	1471973	0.91	0.3647
X11*X23	1.7409361	0.908333	1.92	0.0563
X11*X24	2.9920102	0.908333	3.29	0.0011
X23*X24	25.840941	0.908333	28.45	<.0001
X24*X25	4.5274713	0.908333	4.98	<.0001
X11*X31	-3.444468	0.908333	-3.79	0.0002
X11*X32	-8.610768	0.908333	-9.48	<.0001
X31*X32	18.925932	0.908333	20.84	<.0001
X11*X33	-4.835923	0.908333	-5.32	<.0001
X31*X33	15.411596	0.908333	16.97	<.0001
X32*X33	41.627596	0.908333	45.83	<.0001
X11*X34	-2.273658	0.908333	-2.50	0.0129
X31*X34	3.2788931	0.908333	3.61	0.0004
X32*X34	10.595318	0.908333	11.66	<.0001
X33*X34	12.513428	0.908333	13.78	<.0001
X34*X36	2.4008147	0.908333	2.64	0.0087
X25*X39	-14.62376	0.908333	-16.10	<.0001
X11*X43	-7.266431	0.908333	-8.00	<.0001
X4*X11	8.9549337	0.908333	9.86	<.0001
X4*X23	1.4941426	0.908333	1.64	0.1011
X4*X25	-14.68016	0.908333	-16.16	<.0001
X4	808.76588	447.8614	1.81	0.0720
X4*X4	65.130452	9.874699	6.60	<.0001
X11	-206.4181	776.4456	-0.27	0.7906
X11*X11	44.323444	9.874699	4.49	<.0001
X23	-2551.273	260.3907	-9.80	<.0001
X23*X23	52.752642	9.874699	5.34	<.0001
X24	-1374.363	672.4246	-2.04	0.0419
X24*X24	26.036126	9.874699	2.64	0.0089
X25	-770.189	123.1093	-6.26	<.0001
X25*X25	64.319268	9.874699	6.51	<.0001
X31	-1656.424	280.0714	-5.91	<.0001
X31*X31	29.98537	9.874699	3.04	0.0026
X32	-2021.564	358.5487	-5.64	<.0001

X32*X32	19.56145	9.874699	1.98	0.0486
X33	-2269.759	397.2903	-5.71	<.0001
X33*X33	23.393128	9.874699	2.37	0.0185
X34	-2094.686	420.3747	-4.98	<.0001
X34*X34	33.193372	9.874699	3.36	0.0009
X36	-1657.843	376.1059	-4.41	<.0001
X36*X36	35.352226	9.874699	3.58	0.0004
X39	-1624.574	335.8145	-4.84	<.0001
X39*X39	44.522032	9.874699	4.51	<.0001
X62	-3317.159	4026.074	-0.82	0.4107
X62*X62	20.937604	9.874699	2.12	0.0349
X4*X43	-7.79008	0.908333	-8.58	<.0001
X43	-4501.752	7629.684	-0.59	0.5557
X43*X43	6.1805064	9.874715	0.63	0.5319
X43*X62	5.2905849	10.31668	0.51	0.6085
X32*X36	1.7477877	0.908333	1.92	0.0554
X24*X43	-2.238108	0.908333	-2.46	0.0144

APPENDIX H: 20-Term Quadratic Centered CCD Results.

Response: Response

Summary of Fit

RSquare	0.991196
RSquare Adj	0.989807
Root Mean Square Error	224.1543
Mean of Response	259082.7
Observations (or Sum Wgts)	324

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	44	1578177439	35867669	713.8532
Error	279	14018400.4	50245.16	Prob>F
C Total	323	1592195839	.	<.0001

Lack of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack of Fit	238	11873393	49888.2	0.9536
Pure Error	41	2145008	52317.3	Prob>F
Total Error	279	14018400		0.6009
Max RSq				
0.9987				

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	2321042.2	904587.7	2.57	0.0108
X23*X24	3.3108589	0.560386	5.91	<.0001
X24*X25	1.1674145	0.560386	2.08	0.0381
X11*X32	-4.537755	0.560386	-8.10	<.0001
X31*X32	1.8085418	0.560386	3.23	0.0014
X11*X33	-3.024751	0.560386	-5.40	<.0001
X31*X33	1.0378902	0.560386	1.85	0.0651
X32*X33	5.4214124	0.560386	9.67	<.0001
X31*X34	-1.473253	0.560386	-2.63	0.0090
X32*X34	1.7675876	0.560386	3.15	0.0018
X33*X34	2.8927503	0.560386	5.16	<.0001
X23*X36	-2.010192	0.560386	-3.59	0.0004
X23*X39	1.4024652	0.560386	2.50	0.0129
X11*X43	-7.640161	0.560386	-13.63	<.0001
X36*X62	1.0538908	0.560386	1.88	0.0611
X4*X11	9.4314731	0.560386	16.83	<.0001
X4*X25	-8.148226	0.560386	-14.54	<.0001
X4*X43	-8.056264	0.560386	-14.38	<.0001
X4*X62	1.0557426	0.560386	1.88	0.0606
X4	1463.2532	284.3831	5.15	<.0001
X4*X4	43.642317	6.092085	7.16	<.0001
X11	708.48996	478.5695	1.48	0.1399
X11*X11	31.963091	6.092085	5.25	<.0001
X23	-569.0956	196.4973	-2.90	0.0041
X23*X23	12.657967	6.092085	2.08	0.0386
X24	-443.1499	390.0391	-1.14	0.2569
X24*X24	5.3961512	6.092085	0.89	0.3765
X25	-474.3244	111.4665	-4.26	<.0001
X25*X25	26.269091	6.092085	4.31	<.0001
X31	-308.4809	208.2489	-1.48	0.1397
X31*X31	6.9440391	6.092085	1.14	0.2553
X32	-233.067	257.4288	-0.91	0.3661
X32*X32	3.8464511	6.092085	0.63	0.5283
X33	202.88064	281.613	0.72	0.4719

X33*X33	-7.326207	6.092085	-1.20	0.2302
X34	-1124.084	268.8027	-4.18	<.0001
X34*X34	21.133287	6.092085	3.47	0.0006
X36	-872.4175	268.7081	-3.25	0.0013
X36*X36	18.322487	6.092085	3.01	0.0029
X39	-227.1465	243.8643	-0.93	0.3524
X39*X39	4.6256471	6.092085	0.76	0.4483
X43	-10480.69	4703.147	-2.23	0.0266
X43*X43	14.047911	6.092095	2.31	0.0218
X62	-1018.463	353.6669	-2.88	0.0043
X62*X62	16.274521	6.092085	2.67	0.0080

APPENDIX I: 20-Term Modified Simplex-Lattice Koshal Design Results.

Response: Response

Summary of Fit

RSquare	0.975679
RSquare Adj	0.974213
Root Mean Square Error	17417.16
Mean of Response	282484.3
Observations (or Sum Wgts)	300

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	17	3.43185e12	2.019e11	665.4640
Error	282	8.55469e10	3.0336e8	Prob>F
C Total	299	3.5174e+12		<.0001

Lack of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack of Fit	271	8.55462e10	3.1567e8	5300.927
Pure Error	11	655046.753	59549.7	Prob>F
Total Error	282	8.55469e10		<.0001
Max RSq				
1.0000				

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
X1*X19	-109.5599	2.186617	-50.10	<.0001
X2*X4	-154.1413	20.36459	-7.57	<.0001
X1	1254.7085	28.95107	43.34	<.0001
X2	1229.013	28.99592	42.39	<.0001
X3	1168.1931	28.94112	40.36	<.0001
X4	1259.9121	29.02028	43.41	<.0001
X7	1236.53	28.98149	42.67	<.0001
X10	1214.3189	28.98044	41.90	<.0001
X11	1193.3451	28.97309	41.19	<.0001
X12	1186.906	28.96554	40.98	<.0001
X13	1205.3034	28.9655	41.61	<.0001
X14	1207.0091	28.97622	41.66	<.0001
X15	1192.8421	28.96919	41.18	<.0001
X16	1216.4698	28.97722	41.98	<.0001
X17	1200.8016	28.95631	41.47	<.0001
X18	1221.3663	28.9741	42.15	<.0001
X19	1219.8492	29.01718	42.04	<.0001
X21	1191.1923	28.97175	41.12	<.0001

APPENDIX J: SSN Screening Design Results.

Response: Response

Summary of Fit

RSquare	0.866135
RSquare Adj	0.856882
Root Mean Square Error	1.598977
Mean of Response	14.25722
Observations (or Sum Wgts)	233

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	15	3589.7399	239.316	93.6024
Error	217	554.8101	2.557	Prob>F
C Total	232	4144.5500		<.0001

Lack of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack of Fit	129	490.56735	3.80285	5.2092
Pure Error	88	64.24276	0.73003	Prob>F
Total Error	217	554.81011		<.0001
Max RSq				
0.9845				

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	42.271817	3.342492	12.65	<.0001
X12	-0.046289	0.014626	-3.16	0.0018
X17	-0.045825	0.014626	-3.13	0.0020
X19	-0.042661	0.014626	-2.92	0.0039
X21	-0.062233	0.014626	-4.25	<.0001
X24	-0.074361	0.014626	-5.08	<.0001
X30	-0.043002	0.014626	-2.94	0.0036
X33	-1.36179	0.355138	-3.83	0.0002
X33*X33	0.0294913	0.008064	3.66	0.0003
X40	-0.071224	0.018283	-3.90	0.0001
X41	-0.053037	0.014626	-3.63	0.0004
X43	-0.052874	0.014626	-3.61	0.0004
X53	-1.035472	0.179946	-5.75	<.0001
X53*X53	0.0562222	0.009956	5.65	<.0001
X60	-0.05739	0.020895	-2.75	0.0065
X64	-0.072194	0.014626	-4.94	<.0001

APPENDIX K: SSN CCD Results.

Response: Response

Summary of Fit

RSquare	0.896045
RSquare Adj	0.883586
Root Mean Square Error	0.885606
Mean of Response	12.93567
Observations (or Sum Wgts)	300

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	32	1805.0057	56.4064	71.9197
Error	267	209.4074	0.7843	Prob>F
C Total	299	2014.4130		<.0001

Lack of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack of Fit	250	197.71777	0.790871	1.1501
Pure Error	17	11.68963	0.687625	Prob>F
Total Error	267	209.40739		0.3891
Max RSq				
0.9942				

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	34.404515	0.91822	37.47	<.0001
17	-0.354809	0.091125	-3.89	0.0001
19	-0.106144	0.020675	-5.13	<.0001
21	-0.539014	0.094082	-5.73	<.0001
24	-0.078173	0.007813	-10.01	<.0001
30	-0.059246	0.012661	-4.68	<.0001
33	-0.195804	0.027095	-7.23	<.0001
40	-0.266675	0.041196	-6.47	<.0001
41	-0.10639	0.009575	-11.11	<.0001
43	-0.130725	0.01549	-8.44	<.0001
53	-0.036229	0.023781	-1.52	0.1288
60	-0.139839	0.02028	-6.90	<.0001
17*17	0.0125461	0.004491	2.79	0.0056
21*21	0.0158001	0.004491	3.52	0.0005
12*40	0.0011789	0.000692	1.70	0.0896
12*64	-0.003125	0.000326	-9.60	<.0001
19*30	0.001031	0.000554	1.86	0.0636
21*33	-0.001002	0.000554	-1.81	0.0715
33*40	0.0015349	0.000692	2.22	0.0274
21*41	0.0025412	0.000554	4.59	<.0001
40*41	0.0014065	0.000692	2.03	0.0431
21*43	0.0021281	0.000554	3.84	0.0002
33*43	0.0011151	0.000554	2.01	0.0449
40*43	0.001925	0.000692	2.78	0.0058
17*53	0.0010512	0.000615	1.71	0.0886
30*53	-0.000895	0.000615	-1.45	0.1469
21*60	0.0048547	0.000917	5.29	<.0001
24*60	0.0019155	0.000791	2.42	0.0161
40*60	-0.001205	0.000988	-1.22	0.2240
17*64	0.0015133	0.000554	2.73	0.0067
33*64	0.0033522	0.000554	6.06	<.0001
40*64	0.0016323	0.000692	2.36	0.0190
64	-0.114424	0.016932	-6.76	<.0001

APPENDIX L: Random Number Generator Code and Random Array Arranging Code (C Programming Language).

Random number generator (See Banks *et al.* 1996)

```
{
        rand1 = (seed1*40014)%2147483563;
                if(rand1 < 0) rand1 = 2147483563 + rand1;
        rand2 = (seed2*40692)%2147483399;
                if(rand2 < 0) rand2 = 2147483399 + rand2;
        rand3 = (rand1 - rand2)%2147483562;
                if(rand3 < 0) rand3 = 2147483562 + rand3;
        seed1 = rand1;
        seed2 = rand2;
        return rand3;
}
```

Random array ordering procedure for 400 sample points

```
{
        num = 400;

        /*Fill the arrays*/
        for(i1 = 0; i1 <= 399; i1++)
        {
                ordervec[i1] = i1;  /*Produces array of integers from 0 to 399*/
                randomvec[i1] = 0; /*Produces array of zeros*/
        }

        for(temp = 0; temp <= 399; temp++)
        {
                /*Generates random integer between 0 & num-1*/
                i2 = gen1(i2);
                i4 = i2%num;
                        if(i4 < 0) i4 = num + i4;

                /*Places the number from ordervec[i4] into randomvec[temp]*/
                randomvec[temp] = ordervec[i4];

                /*Moves every element in ordervec (below the number drawn) up 1*/
                for(i3 = i4; i3 <= num-1; i3++)
                {
                        if(i3!=399) ordervec[i3] = ordervec[i3+1];
                }

                /*Decrements num, but never to zero*/
                if(num > 1)        num--;
        }

return;
}
```

APPENDIX M: Sampling Pseudocode.

Initialize:

For each random variate, n samples at each point, need three arrays of size n, one with the stochastic values and two of zeros. The number of each stochastic value of probability p is found by multiplying $p*n$. Non-integer values are dealt with using the McKay 1988 procedure (also shown in Bailey 1995).

A file of decision variable values (decval.prn) is produced, with each row containing a complete set of values to sample space. A separate file (rnum.prn) contains random number seeds and number of samples (rows) in decision variable file.

Sampling:

```
Read MPS problem input file into solver
Read random number seeds and number of samples from rnum.prn.
Assign i = number of samples
    For loop counter = 0 to loop counter = i-1
        Read next row of decision variable values from decval.prn
        Change decision variable values in solver
            For loop counter 2 = 0 to loop counter 2 = n-1
                Arrange random vector (see Appendix L)
                Arrange stochastic vectors in random order of random vector
                Change stochastic values in solver
                Optimize problem with current values
                Store solution value
                End
            Compute average of values
            Print average to file
            End
        End
    End
End
```

APPENDIX N: 20-Term CCD Design Matrix.

Run	X4	X11	X23	X24	X25	X31	X32	X33	X34	X36	X39	X43	X62
1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	-1	1	1	-1	-1	-1
3	1	1	1	1	1	1	-1	1	1	1	-1	1	1
4	1	1	1	1	1	1	-1	-1	1	1	1	-1	-1
5	1	1	1	1	1	-1	1	1	1	-1	-1	-1	-1
6	1	1	1	1	1	-1	1	-1	1	-1	1	1	1
7	1	1	1	1	1	-1	-1	1	1	-1	1	-1	-1
8	1	1	1	1	1	-1	-1	-1	1	-1	-1	1	1
9	1	1	1	1	-1	1	1	1	1	-1	-1	1	1
10	1	1	1	1	-1	1	1	-1	1	-1	1	-1	-1
11	1	1	1	1	-1	1	-1	1	1	-1	1	1	1
12	1	1	1	1	-1	1	-1	-1	1	-1	-1	-1	-1
13	1	1	1	1	-1	-1	1	1	1	1	1	-1	-1
14	1	1	1	1	-1	-1	1	-1	1	1	-1	1	1
15	1	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1
16	1	1	1	1	-1	-1	-1	-1	1	1	1	1	1
17	1	1	1	-1	1	1	1	1	-1	-1	1	-1	1
18	1	1	1	-1	1	1	1	-1	-1	-1	-1	1	-1
19	1	1	1	-1	1	1	-1	1	-1	-1	-1	-1	1
20	1	1	1	-1	1	1	-1	-1	-1	-1	1	1	-1
21	1	1	1	-1	1	-1	1	1	-1	1	-1	1	-1
22	1	1	1	-1	1	-1	1	-1	-1	1	1	-1	1
23	1	1	1	-1	1	-1	-1	1	-1	1	1	1	-1
24	1	1	1	-1	1	-1	-1	-1	-1	1	-1	-1	1
25	1	1	1	-1	-1	1	1	1	-1	1	-1	-1	1
26	1	1	1	-1	-1	1	1	-1	-1	1	1	1	-1
27	1	1	1	-1	-1	1	-1	1	-1	1	1	-1	1
28	1	1	1	-1	-1	1	-1	-1	-1	1	-1	1	-1
29	1	1	1	-1	-1	-1	1	1	-1	-1	1	1	-1
30	1	1	1	-1	-1	-1	1	-1	-1	-1	-1	-1	1
31	1	1	1	-1	-1	-1	-1	1	-1	-1	-1	1	-1
32	1	1	1	-1	-1	-1	-1	-1	-1	-1	1	-1	1
33	1	1	-1	1	1	1	1	1	-1	-1	1	1	-1
34	1	1	-1	1	1	1	1	-1	-1	-1	-1	-1	1
35	1	1	-1	1	1	1	-1	1	-1	-1	-1	1	-1
36	1	1	-1	1	1	1	-1	-1	-1	-1	1	-1	1
37	1	1	-1	1	1	-1	1	1	-1	1	-1	-1	1
38	1	1	-1	1	1	-1	1	-1	-1	1	1	1	-1
39	1	1	-1	1	1	-1	-1	1	-1	1	1	-1	1
40	1	1	-1	1	1	-1	-1	-1	-1	1	-1	1	-1
41	1	1	-1	1	-1	1	1	1	-1	1	-1	1	-1
42	1	1	-1	1	-1	1	1	-1	-1	1	1	-1	1
43	1	1	-1	1	-1	1	-1	1	-1	1	1	1	-1

44	1	1	-1	1	-1	1	-1	-1	-1	1	-1	-1	1
45	1	1	-1	1	-1	-1	1	1	-1	-1	1	-1	1
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274	0	0	0	0	0	0	0	0	-1	0	0	0	0
275	0	0	0	0	0	0	0	0	0	1	0	0	0
276	0	0	0	0	0	0	0	0	0	-1	0	0	0
277	0	0	0	0	0	0	0	0	0	0	1	0	0
278	0	0	0	0	0	0	0	0	0	0	-1	0	0
279	0	0	0	0	0	0	0	0	0	0	0	1	0
280	0	0	0	0	0	0	0	0	0	0	0	-1	0
281	0	0	0	0	0	0	0	0	0	0	0	0	1
282	0	0	0	0	0	0	0	0	0	0	0	0	-1
283	0	0	0	0	0	0	0	0	0	0	0	0	0
284	0	0	0	0	0	0	0	0	0	0	0	0	0
285	0	0	0	0	0	0	0	0	0	0	0	0	0
286	0	0	0	0	0	0	0	0	0	0	0	0	0
287	0	0	0	0	0	0	0	0	0	0	0	0	0
288	0	0	0	0	0	0	0	0	0	0	0	0	0
289	0	0	0	0	0	0	0	0	0	0	0	0	0
290	0	0	0	0	0	0	0	0	0	0	0	0	0
291	0	0	0	0	0	0	0	0	0	0	0	0	0
292	0	0	0	0	0	0	0	0	0	0	0	0	0
293	0	0	0	0	0	0	0	0	0	0	0	0	0

294	0	0	0	0	0	0	0	0	0	0	0	0	0
295	0	0	0	0	0	0	0	0	0	0	0	0	0
296	0	0	0	0	0	0	0	0	0	0	0	0	0
297	0	0	0	0	0	0	0	0	0	0	0	0	0
298	0	0	0	0	0	0	0	0	0	0	0	0	0
299	0	0	0	0	0	0	0	0	0	0	0	0	0
300	0	0	0	0	0	0	0	0	0	0	0	0	0
301	0	0	0	0	0	0	0	0	0	0	0	0	0
302	0	0	0	0	0	0	0	0	0	0	0	0	0
303	0	0	0	0	0	0	0	0	0	0	0	0	0
304	0	0	0	0	0	0	0	0	0	0	0	0	0
305	0	0	0	0	0	0	0	0	0	0	0	0	0
306	0	0	0	0	0	0	0	0	0	0	0	0	0
307	0	0	0	0	0	0	0	0	0	0	0	0	0
308	0	0	0	0	0	0	0	0	0	0	0	0	0
309	0	0	0	0	0	0	0	0	0	0	0	0	0
310	0	0	0	0	0	0	0	0	0	0	0	0	0
311	0	0	0	0	0	0	0	0	0	0	0	0	0
312	0	0	0	0	0	0	0	0	0	0	0	0	0
313	0	0	0	0	0	0	0	0	0	0	0	0	0
314	0	0	0	0	0	0	0	0	0	0	0	0	0
315	0	0	0	0	0	0	0	0	0	0	0	0	0
316	0	0	0	0	0	0	0	0	0	0	0	0	0
317	0	0	0	0	0	0	0	0	0	0	0	0	0
318	0	0	0	0	0	0	0	0	0	0	0	0	0
319	0	0	0	0	0	0	0	0	0	0	0	0	0
320	0	0	0	0	0	0	0	0	0	0	0	0	0
321	0	0	0	0	0	0	0	0	0	0	0	0	0
322	0	0	0	0	0	0	0	0	0	0	0	0	0
323	0	0	0	0	0	0	0	0	0	0	0	0	0
324	0	0	0	0	0	0	0	0	0	0	0	0	0

APPENDIX O: 20-Term Minimal Bias Design Matrix.

RUN	X4	X11	X23	X24	X25	X31	X32	X33	X34	X36	X39	X43	X62
1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	-1	1	1	-1	-1	-1
3	1	1	1	1	1	1	-1	1	1	1	-1	1	1
4	1	1	1	1	1	1	-1	-1	1	1	1	-1	-1
5	1	1	1	1	1	-1	1	1	1	-1	-1	-1	-1
6	1	1	1	1	1	-1	1	-1	1	-1	1	1	1
7	1	1	1	1	1	-1	-1	1	1	-1	1	-1	-1
8	1	1	1	1	1	-1	-1	-1	1	-1	-1	1	1
9	1	1	1	1	-1	1	1	1	1	-1	-1	1	1
10	1	1	1	1	-1	1	1	-1	1	-1	1	-1	-1
11	1	1	1	1	-1	1	-1	1	1	-1	1	1	1
12	1	1	1	1	-1	1	-1	-1	1	-1	-1	-1	-1
13	1	1	1	1	-1	-1	1	1	1	1	1	-1	-1
14	1	1	1	1	-1	-1	1	-1	1	1	-1	1	1
15	1	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1
16	1	1	1	1	-1	-1	-1	-1	1	1	1	1	1
17	1	1	1	-1	1	1	1	1	-1	-1	1	-1	1
18	1	1	1	-1	1	1	1	-1	-1	-1	-1	1	-1
19	1	1	1	-1	1	1	-1	1	-1	-1	-1	-1	1
20	1	1	1	-1	1	1	-1	-1	-1	-1	1	1	-1
21	1	1	1	-1	1	-1	1	1	-1	1	-1	1	-1
22	1	1	1	-1	1	-1	1	-1	-1	1	1	-1	1
23	1	1	1	-1	1	-1	-1	1	-1	1	1	1	-1
24	1	1	1	-1	1	-1	-1	-1	-1	1	-1	-1	1
25	1	1	1	-1	-1	1	1	1	-1	1	-1	-1	1
26	1	1	1	-1	-1	1	1	-1	-1	1	1	1	-1
27	1	1	1	-1	-1	1	-1	1	-1	1	1	-1	1
28	1	1	1	-1	-1	1	-1	-1	-1	1	-1	1	-1
29	1	1	1	-1	-1	-1	1	1	-1	-1	1	1	-1
30	1	1	1	-1	-1	-1	1	-1	-1	-1	-1	-1	1
31	1	1	1	-1	-1	-1	-1	1	-1	-1	-1	1	-1
32	1	1	1	-1	-1	-1	-1	-1	-1	-1	1	-1	1
33	1	1	-1	1	1	1	1	1	-1	-1	1	1	-1
34	1	1	-1	1	1	1	1	-1	-1	-1	-1	-1	1
35	1	1	-1	1	1	1	-1	1	-1	-1	-1	1	-1
36	1	1	-1	1	1	1	-1	-1	-1	-1	1	-1	1
37	1	1	-1	1	1	-1	1	1	-1	1	-1	-1	1
38	1	1	-1	1	1	-1	1	-1	-1	1	1	1	-1
39	1	1	-1	1	1	-1	-1	1	-1	1	1	-1	1
40	1	1	-1	1	1	-1	-1	-1	-1	1	-1	1	-1
41	1	1	-1	1	-1	1	1	1	-1	1	-1	1	-1
42	1	1	-1	1	-1	1	1	-1	-1	1	1	-1	1
43	1	1	-1	1	-1	1	-1	1	-1	1	1	1	-1

44	1	1	-1	1	-1	1	-1	-1	-1	1	-1	-1	1
45	1	1	-1	1	-1	-1	1	1	-1	-1	1	-1	1
46	1	1	-1	1	-1	-1	1	-1	-1	-1	-1	1	-1
47	1	1	-1	1	-1	-1	-1	1	-1	-1	-1	-1	1
48	1	1	-1	1	-1	-1	-1	-1	-1	-1	1	1	-1
49	1	1	-1	-1	1	1	1	1	1	1	1	-1	-1
50	1	1	-1	-1	1	1	1	-1	1	1	-1	1	1
51	1	1	-1	-1	1	1	-1	1	1	1	-1	-1	-1
52	1	1	-1	-1	1	1	-1	-1	1	1	1	1	1
53	1	1	-1	-1	1	-1	1	1	1	-1	-1	1	1
54	1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	-1
55	1	1	-1	-1	1	-1	-1	1	1	-1	1	1	1
56	1	1	-1	-1	1	-1	-1	-1	1	-1	-1	-1	-1
57	1	1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1
58	1	1	-1	-1	-1	1	1	-1	1	-1	1	1	1
59	1	1	-1	-1	-1	1	-1	1	1	-1	1	-1	-1
60	1	1	-1	-1	-1	1	-1	-1	1	-1	-1	1	1
61	1	1	-1	-1	-1	-1	1	1	1	1	1	1	1
62	1	1	-1	-1	-1	-1	1	-1	1	1	-1	-1	-1
63	1	1	-1	-1	-1	-1	-1	1	1	1	-1	1	1
64	1	1	-1	-1	-1	-1	-1	-1	1	1	1	-1	-1
65	1	-1	1	1	1	1	1	1	-1	1	1	-1	1
66	1	-1	1	1	1	1	1	-1	-1	1	-1	1	-1
67	1	-1	1	1	1	1	-1	1	-1	1	-1	-1	1
68	1	-1	1	1	1	1	-1	-1	-1	1	1	1	-1
69	1	-1	1	1	1	-1	1	1	-1	-1	-1	1	-1
70	1	-1	1	1	1	-1	1	-1	-1	-1	1	-1	1
71	1	-1	1	1	1	-1	-1	1	-1	-1	1	1	-1
72	1	-1	1	1	1	-1	-1	-1	-1	-1	-1	-1	1
73	1	-1	1	1	-1	1	1	1	-1	-1	-1	-1	1
74	1	-1	1	1	-1	1	1	-1	-1	-1	1	1	-1
75	1	-1	1	1	-1	1	-1	1	-1	-1	1	-1	1
76	1	-1	1	1	-1	1	-1	-1	-1	-1	-1	1	-1
77	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1
78	1	-1	1	1	-1	-1	1	-1	-1	1	-1	-1	1
79	1	-1	1	1	-1	-1	-1	1	-1	1	-1	1	-1
80	1	-1	1	1	-1	-1	-1	-1	-1	1	1	-1	1
81	1	-1	1	-1	1	1	1	1	1	-1	1	1	1
82	1	-1	1	-1	1	1	1	-1	1	-1	-1	-1	-1
83	1	-1	1	-1	1	1	-1	1	1	-1	-1	1	1
84	1	-1	1	-1	1	1	-1	-1	1	-1	1	-1	-1
85	1	-1	1	-1	1	-1	1	1	1	1	-1	-1	-1
86	1	-1	1	-1	1	-1	1	-1	1	1	1	1	1
87	1	-1	1	-1	1	-1	-1	1	1	1	1	-1	-1
88	1	-1	1	-1	1	-1	-1	-1	1	1	-1	1	1
89	1	-1	1	-1	-1	1	1	1	1	1	-1	1	1
90	1	-1	1	-1	-1	1	1	-1	1	1	1	-1	-1
91	1	-1	1	-1	-1	1	-1	1	1	1	1	1	1
92	1	-1	1	-1	-1	1	-1	-1	1	1	-1	-1	-1
93	1	-1	1	-1	-1	-1	1	1	1	-1	1	-1	-1

94	1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1
95	1	-1	1	-1	-1	-1	-1	1	1	-1	-1	-1	-1
96	1	-1	1	-1	-1	-1	-1	-1	1	-1	1	1	1
97	1	-1	-1	1	1	1	1	1	1	-1	1	-1	-1
98	1	-1	-1	1	1	1	1	-1	1	-1	-1	1	1
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100	1	-1	-1	1	1	1	-1	-1	1	-1	1	1	1
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102	1	-1	-1	1	1	-1	1	-1	1	1	1	-1	-1
103	1	-1	-1	1	1	-1	-1	1	1	1	1	1	1
104	1	-1	-1	1	1	-1	-1	-1	1	1	-1	-1	-1
105	1	-1	-1	1	-1	1	1	1	1	1	-1	-1	-1
106	1	-1	-1	1	-1	1	1	-1	1	1	1	1	1
107	1	-1	-1	1	-1	1	-1	1	1	1	1	-1	-1
108	1	-1	-1	1	-1	1	-1	-1	1	1	-1	1	1
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110	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	-1	-1
111	1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1
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117	1	-1	-1	-1	1	-1	1	1	-1	-1	-1	-1	1
118	1	-1	-1	-1	1	-1	1	-1	-1	-1	1	1	-1
119	1	-1	-1	-1	1	-1	-1	1	-1	-1	1	-1	1
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121	1	-1	-1	-1	-1	1	1	1	-1	-1	-1	1	-1
122	1	-1	-1	-1	-1	1	1	-1	-1	-1	1	-1	1
123	1	-1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1
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125	1	-1	-1	-1	-1	-1	1	1	-1	1	1	-1	1
126	1	-1	-1	-1	-1	-1	1	-1	-1	1	-1	1	-1
127	1	-1	-1	-1	-1	-1	-1	1	-1	1	-1	-1	1
128	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	-1
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146	-1	1	1	-1	1	1	1	-1	1	-1	-1	1	1
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149	-1	1	1	-1	1	-1	1	1	1	1	-1	1	1
150	-1	1	1	-1	1	-1	1	-1	1	1	1	-1	-1
151	-1	1	1	-1	1	-1	-1	1	1	1	1	1	1
152	-1	1	1	-1	1	-1	-1	-1	1	1	-1	-1	-1
153	-1	1	1	-1	-1	1	1	1	1	1	-1	-1	-1
154	-1	1	1	-1	-1	1	1	-1	1	1	1	1	1
155	-1	1	1	-1	-1	1	-1	1	1	1	1	-1	-1
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161	-1	1	-1	1	1	1	1	1	1	-1	1	1	1
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163	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1
164	-1	1	-1	1	1	1	-1	-1	1	-1	1	-1	-1
165	-1	1	-1	1	1	-1	1	1	1	1	-1	-1	-1
166	-1	1	-1	1	1	-1	1	-1	1	1	1	1	1
167	-1	1	-1	1	1	-1	-1	1	1	1	1	-1	-1
168	-1	1	-1	1	1	-1	-1	-1	1	1	-1	1	1
169	-1	1	-1	1	-1	1	1	1	1	1	-1	1	1
170	-1	1	-1	1	-1	1	1	-1	1	1	1	-1	-1
171	-1	1	-1	1	-1	1	-1	1	1	1	1	1	1
172	-1	1	-1	1	-1	1	-1	-1	1	1	-1	-1	-1
173	-1	1	-1	1	-1	-1	1	1	1	-1	1	-1	-1
174	-1	1	-1	1	-1	-1	1	-1	1	-1	-1	1	1
175	-1	1	-1	1	-1	-1	-1	1	1	-1	-1	-1	-1
176	-1	1	-1	1	-1	-1	-1	-1	1	-1	1	1	1
177	-1	1	-1	-1	1	1	1	1	-1	1	1	-1	1
178	-1	1	-1	-1	1	1	1	-1	-1	1	-1	1	-1
179	-1	1	-1	-1	1	1	-1	1	-1	1	-1	-1	1
180	-1	1	-1	-1	1	1	-1	-1	-1	1	1	1	-1
181	-1	1	-1	-1	1	-1	1	1	-1	-1	-1	1	-1
182	-1	1	-1	-1	1	-1	1	-1	-1	-1	1	-1	1
183	-1	1	-1	-1	1	-1	-1	1	-1	-1	1	1	-1
184	-1	1	-1	-1	1	-1	-1	-1	-1	-1	-1	-1	1
185	-1	1	-1	-1	-1	1	1	1	-1	-1	-1	-1	1
186	-1	1	-1	-1	-1	1	1	-1	-1	-1	1	1	-1
187	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	-1	1
188	-1	1	-1	-1	-1	1	-1	-1	-1	-1	-1	1	-1
189	-1	1	-1	-1	-1	-1	1	1	-1	1	1	1	-1
190	-1	1	-1	-1	-1	-1	1	-1	-1	1	-1	-1	1
191	-1	1	-1	-1	-1	-1	-1	1	-1	1	-1	1	-1
192	-1	1	-1	-1	-1	-1	-1	-1	-1	1	1	-1	1
193	-1	-1	1	1	1	1	1	1	1	1	1	-1	-1

194	-1	-1	1	1	1	1	1	-1	1	1	-1	1	1
195	-1	-1	1	1	1	1	-1	1	1	1	-1	-1	-1
196	-1	-1	1	1	1	1	-1	-1	1	1	1	1	1
197	-1	-1	1	1	1	-1	1	1	1	-1	-1	1	1
198	-1	-1	1	1	1	-1	1	-1	1	-1	1	-1	-1
199	-1	-1	1	1	1	-1	-1	1	1	-1	1	1	1
200	-1	-1	1	1	1	-1	-1	-1	1	-1	-1	-1	-1
201	-1	-1	1	1	-1	1	1	1	1	-1	-1	-1	-1
202	-1	-1	1	1	-1	1	1	-1	1	-1	1	1	1
203	-1	-1	1	1	-1	1	-1	1	1	-1	1	-1	-1
204	-1	-1	1	1	-1	1	-1	-1	1	-1	-1	1	1
205	-1	-1	1	1	-1	-1	1	1	1	1	1	1	1
206	-1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	-1
207	-1	-1	1	1	-1	-1	-1	1	1	1	-1	1	1
208	-1	-1	1	1	-1	-1	-1	-1	1	1	1	-1	-1
209	-1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1
210	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1
211	-1	-1	1	-1	1	1	-1	1	-1	-1	-1	1	-1
212	-1	-1	1	-1	1	1	-1	-1	-1	-1	1	-1	1
213	-1	-1	1	-1	1	-1	1	1	-1	1	-1	-1	1
214	-1	-1	1	-1	1	-1	1	-1	-1	1	1	1	-1
215	-1	-1	1	-1	1	-1	-1	1	-1	1	1	-1	1
216	-1	-1	1	-1	1	-1	-1	-1	-1	1	-1	1	-1
217	-1	-1	1	-1	-1	1	1	1	-1	1	-1	1	-1
218	-1	-1	1	-1	-1	1	1	-1	-1	1	1	-1	1
219	-1	-1	1	-1	-1	1	-1	1	-1	1	1	1	-1
220	-1	-1	1	-1	-1	1	-1	-1	-1	1	-1	-1	1
221	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	-1	1
222	-1	-1	1	-1	-1	-1	1	-1	-1	-1	-1	1	-1
223	-1	-1	1	-1	-1	-1	-1	1	-1	-1	-1	-1	1
224	-1	-1	1	-1	-1	-1	-1	-1	-1	-1	1	1	-1
225	-1	-1	-1	1	1	1	1	1	-1	-1	1	-1	1
226	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	-1
227	-1	-1	-1	1	1	1	-1	1	-1	-1	-1	-1	1
228	-1	-1	-1	1	1	1	-1	-1	-1	-1	1	1	-1
229	-1	-1	-1	1	1	-1	1	1	-1	1	-1	1	-1
230	-1	-1	-1	1	1	-1	1	-1	-1	1	1	-1	1
231	-1	-1	-1	1	1	-1	-1	1	-1	1	1	1	-1
232	-1	-1	-1	1	1	-1	-1	-1	-1	1	-1	-1	1
233	-1	-1	-1	1	-1	1	1	1	-1	1	-1	-1	1
234	-1	-1	-1	1	-1	1	1	-1	-1	1	1	1	-1
235	-1	-1	-1	1	-1	1	-1	1	-1	1	1	-1	1
236	-1	-1	-1	1	-1	1	-1	-1	-1	1	-1	1	-1
237	-1	-1	-1	1	-1	-1	1	1	-1	-1	1	1	-1
238	-1	-1	-1	1	-1	-1	1	-1	-1	-1	-1	-1	1
239	-1	-1	-1	1	-1	-1	-1	1	-1	-1	-1	1	-1
240	-1	-1	-1	1	-1	-1	-1	-1	-1	-1	1	-1	1
241	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
242	-1	-1	-1	-1	1	1	1	-1	1	1	-1	-1	-1
243	-1	-1	-1	-1	1	1	-1	1	1	1	-1	1	1

244	-1	-1	-1	-1	1	1	-1	-1	1	1	1	-1	-1
245	-1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1
246	-1	-1	-1	-1	1	-1	1	-1	1	-1	1	1	1
247	-1	-1	-1	-1	1	-1	-1	1	1	-1	1	-1	-1
248	-1	-1	-1	-1	1	-1	-1	-1	1	-1	-1	1	1
249	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	1	1
250	-1	-1	-1	-1	-1	1	1	-1	1	-1	1	-1	-1
251	-1	-1	-1	-1	-1	1	-1	1	1	-1	1	1	1
252	-1	-1	-1	-1	-1	1	-1	-1	1	-1	-1	-1	-1
253	-1	-1	-1	-1	-1	-1	1	1	1	1	1	-1	-1
254	-1	-1	-1	-1	-1	-1	1	-1	1	1	-1	1	1
255	-1	-1	-1	-1	-1	-1	-1	1	1	1	-1	-1	-1
256	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1
257	1	0	0	0	0	0	0	0	0	0	0	0	0
258	-1	0	0	0	0	0	0	0	0	0	0	0	0
259	0	1	0	0	0	0	0	0	0	0	0	0	0
260	0	-1	0	0	0	0	0	0	0	0	0	0	0
261	0	0	1	0	0	0	0	0	0	0	0	0	0
262	0	0	-1	0	0	0	0	0	0	0	0	0	0
263	0	0	0	1	0	0	0	0	0	0	0	0	0
264	0	0	0	-1	0	0	0	0	0	0	0	0	0
265	0	0	0	0	1	0	0	0	0	0	0	0	0
266	0	0	0	0	-1	0	0	0	0	0	0	0	0
267	0	0	0	0	0	1	0	0	0	0	0	0	0
268	0	0	0	0	0	-1	0	0	0	0	0	0	0
269	0	0	0	0	0	0	1	0	0	0	0	0	0
270	0	0	0	0	0	0	-1	0	0	0	0	0	0
271	0	0	0	0	0	0	0	1	0	0	0	0	0
272	0	0	0	0	0	0	0	-1	0	0	0	0	0
273	0	0	0	0	0	0	0	0	1	0	0	0	0
274	0	0	0	0	0	0	0	0	-1	0	0	0	0
275	0	0	0	0	0	0	0	0	0	1	0	0	0
276	0	0	0	0	0	0	0	0	0	-1	0	0	0
277	0	0	0	0	0	0	0	0	0	0	1	0	0
278	0	0	0	0	0	0	0	0	0	0	-1	0	0
279	0	0	0	0	0	0	0	0	0	0	0	1	0
280	0	0	0	0	0	0	0	0	0	0	0	-1	0
281	0	0	0	0	0	0	0	0	0	0	0	0	1
282	0	0	0	0	0	0	0	0	0	0	0	0	-1
283	0	0	0	0	0	0	0	0	0	0	0	0	0
284	0	0	0	0	0	0	0	0	0	0	0	0	0
285	0	0	0	0	0	0	0	0	0	0	0	0	0
286	0	0	0	0	0	0	0	0	0	0	0	0	0
287	0	0	0	0	0	0	0	0	0	0	0	0	0
288	0	0	0	0	0	0	0	0	0	0	0	0	0
289	0	0	0	0	0	0	0	0	0	0	0	0	0
290	0	0	0	0	0	0	0	0	0	0	0	0	0
291	0	0	0	0	0	0	0	0	0	0	0	0	0
292	0	0	0	0	0	0	0	0	0	0	0	0	0
293	0	0	0	0	0	0	0	0	0	0	0	0	0

294	0	0	0	0	0	0	0	0	0	0	0	0	0
295	0	0	0	0	0	0	0	0	0	0	0	0	0
296	0	0	0	0	0	0	0	0	0	0	0	0	0
297	0	0	0	0	0	0	0	0	0	0	0	0	0
298	0	0	0	0	0	0	0	0	0	0	0	0	0
299	0	0	0	0	0	0	0	0	0	0	0	0	0
300	0	0	0	0	0	0	0	0	0	0	0	0	0
301	0	0	0	0	0	0	0	0	0	0	0	0	0
302	0	0	0	0	0	0	0	0	0	0	0	0	0
303	0	0	0	0	0	0	0	0	0	0	0	0	0
304	0	0	0	0	0	0	0	0	0	0	0	0	0
305	0	0	0	0	0	0	0	0	0	0	0	0	0
306	0	0	0	0	0	0	0	0	0	0	0	0	0
307	0	0	0	0	0	0	0	0	0	0	0	0	0
308	0	0	0	0	0	0	0	0	0	0	0	0	0
309	0	0	0	0	0	0	0	0	0	0	0	0	0
310	0	0	0	0	0	0	0	0	0	0	0	0	0
311	0	0	0	0	0	0	0	0	0	0	0	0	0
312	0	0	0	0	0	0	0	0	0	0	0	0	0
313	0	0	0	0	0	0	0	0	0	0	0	0	0
314	0	0	0	0	0	0	0	0	0	0	0	0	0
315	0	0	0	0	0	0	0	0	0	0	0	0	0
316	0	0	0	0	0	0	0	0	0	0	0	0	0
317	0	0	0	0	0	0	0	0	0	0	0	0	0
318	0	0	0	0	0	0	0	0	0	0	0	0	0
319	0	0	0	0	0	0	0	0	0	0	0	0	0
320	0	0	0	0	0	0	0	0	0	0	0	0	0
321	0	0	0	0	0	0	0	0	0	0	0	0	0
322	0	0	0	0	0	0	0	0	0	0	0	0	0
323	0	0	0	0	0	0	0	0	0	0	0	0	0
324	0	0	0	0	0	0	0	0	0	0	0	0	0

APPENDIX P: 20-Term MSLK Deviation Matrix.

Axial points not included

run	X1	X2	X3	X4	X7	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19	X21
1	-5	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	-5	0	5	0	0	0	0	0	0	0	0	0	0	0	0	0
3	-5	0	0	5	0	0	0	0	0	0	0	0	0	0	0	0
4	-5	0	0	0	5	0	0	0	0	0	0	0	0	0	0	0
5	-5	0	0	0	0	5	0	0	0	0	0	0	0	0	0	0
6	-5	0	0	0	0	0	5	0	0	0	0	0	0	0	0	0
7	-5	0	0	0	0	0	0	5	0	0	0	0	0	0	0	0
8	-5	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0
9	-5	0	0	0	0	0	0	0	0	5	0	0	0	0	0	0
10	-5	0	0	0	0	0	0	0	0	0	5	0	0	0	0	0
11	-5	0	0	0	0	0	0	0	0	0	0	5	0	0	0	0
12	-5	0	0	0	0	0	0	0	0	0	0	0	5	0	0	0
13	-5	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0
14	-5	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0
15	-5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5
16	0	-5	5	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	-5	0	5	0	0	0	0	0	0	0	0	0	0	0	0
18	0	-5	0	0	5	0	0	0	0	0	0	0	0	0	0	0
19	0	-5	0	0	0	5	0	0	0	0	0	0	0	0	0	0
20	0	-5	0	0	0	0	5	0	0	0	0	0	0	0	0	0
21	0	-5	0	0	0	0	0	5	0	0	0	0	0	0	0	0
22	0	-5	0	0	0	0	0	0	5	0	0	0	0	0	0	0
23	0	-5	0	0	0	0	0	0	0	5	0	0	0	0	0	0
24	0	-5	0	0	0	0	0	0	0	0	5	0	0	0	0	0
25	0	-5	0	0	0	0	0	0	0	0	0	5	0	0	0	0
26	0	-5	0	0	0	0	0	0	0	0	0	0	5	0	0	0
27	0	-5	0	0	0	0	0	0	0	0	0	0	0	5	0	0
28	0	-5	0	0	0	0	0	0	0	0	0	0	0	0	5	0
29	0	-5	0	0	0	0	0	0	0	0	0	0	0	0	0	5
30	0	0	-5	5	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	-5	0	5	0	0	0	0	0	0	0	0	0	0	0
32	0	0	-5	0	0	5	0	0	0	0	0	0	0	0	0	0
33	0	0	-5	0	0	0	5	0	0	0	0	0	0	0	0	0
34	0	0	-5	0	0	0	0	5	0	0	0	0	0	0	0	0
35	0	0	-5	0	0	0	0	0	5	0	0	0	0	0	0	0
36	0	0	-5	0	0	0	0	0	0	5	0	0	0	0	0	0
37	0	0	-5	0	0	0	0	0	0	0	5	0	0	0	0	0
38	0	0	-5	0	0	0	0	0	0	0	0	5	0	0	0	0
39	0	0	-5	0	0	0	0	0	0	0	0	0	5	0	0	0
40	0	0	-5	0	0	0	0	0	0	0	0	0	0	5	0	0
41	0	0	-5	0	0	0	0	0	0	0	0	0	0	0	5	0
42	0	0	-5	0	0	0	0	0	0	0	0	0	0	0	0	5
43	0	0	0	-5	5	0	0	0	0	0	0	0	0	0	0	0
44	0	0	0	-5	0	5	0	0	0	0	0	0	0	0	0	0

45	0	0	0	-5	0	0	5	0	0	0	0	0	0	0	0	0
46	0	0	0	-5	0	0	0	5	0	0	0	0	0	0	0	0
47	0	0	0	-5	0	0	0	0	5	0	0	0	0	0	0	0
48	0	0	0	-5	0	0	0	0	0	5	0	0	0	0	0	0
49	0	0	0	-5	0	0	0	0	0	0	5	0	0	0	0	0
50	0	0	0	-5	0	0	0	0	0	0	0	5	0	0	0	0
51	0	0	0	-5	0	0	0	0	0	0	0	0	5	0	0	0
52	0	0	0	-5	0	0	0	0	0	0	0	0	0	5	0	0
53	0	0	0	-5	0	0	0	0	0	0	0	0	0	0	5	0
54	0	0	0	-5	0	0	0	0	0	0	0	0	0	0	0	5
55	0	0	0	0	-5	5	0	0	0	0	0	0	0	0	0	0
56	0	0	0	0	-5	0	5	0	0	0	0	0	0	0	0	0
57	0	0	0	0	-5	0	0	5	0	0	0	0	0	0	0	0
58	0	0	0	0	-5	0	0	0	5	0	0	0	0	0	0	0
59	0	0	0	0	-5	0	0	0	0	5	0	0	0	0	0	0
60	0	0	0	0	-5	0	0	0	0	0	5	0	0	0	0	0
61	0	0	0	0	-5	0	0	0	0	0	0	5	0	0	0	0
62	0	0	0	0	-5	0	0	0	0	0	0	0	5	0	0	0
63	0	0	0	0	-5	0	0	0	0	0	0	0	0	5	0	0
64	0	0	0	0	-5	0	0	0	0	0	0	0	0	0	5	0
65	0	0	0	0	-5	0	0	0	0	0	0	0	0	0	0	5
66	0	0	0	0	0	-5	5	0	0	0	0	0	0	0	0	0
67	0	0	0	0	0	-5	0	5	0	0	0	0	0	0	0	0
68	0	0	0	0	0	-5	0	0	5	0	0	0	0	0	0	0
69	0	0	0	0	0	-5	0	0	0	5	0	0	0	0	0	0
70	0	0	0	0	0	-5	0	0	0	0	5	0	0	0	0	0
71	0	0	0	0	0	-5	0	0	0	0	0	5	0	0	0	0
72	0	0	0	0	0	-5	0	0	0	0	0	0	5	0	0	0
73	0	0	0	0	0	-5	0	0	0	0	0	0	0	5	0	0
74	0	0	0	0	0	-5	0	0	0	0	0	0	0	0	5	0
75	0	0	0	0	0	-5	0	0	0	0	0	0	0	0	0	5
76	0	0	0	0	0	0	-5	5	0	0	0	0	0	0	0	0
77	0	0	0	0	0	0	-5	0	5	0	0	0	0	0	0	0
78	0	0	0	0	0	0	-5	0	0	5	0	0	0	0	0	0
79	0	0	0	0	0	0	-5	0	0	0	5	0	0	0	0	0
80	0	0	0	0	0	0	-5	0	0	0	0	5	0	0	0	0
81	0	0	0	0	0	0	-5	0	0	0	0	0	5	0	0	0
82	0	0	0	0	0	0	-5	0	0	0	0	0	0	5	0	0
83	0	0	0	0	0	0	-5	0	0	0	0	0	0	0	5	0
84	0	0	0	0	0	0	-5	0	0	0	0	0	0	0	0	5
85	0	0	0	0	0	0	0	-5	5	0	0	0	0	0	0	0
86	0	0	0	0	0	0	0	-5	0	5	0	0	0	0	0	0
87	0	0	0	0	0	0	0	-5	0	0	5	0	0	0	0	0
88	0	0	0	0	0	0	0	-5	0	0	0	5	0	0	0	0
89	0	0	0	0	0	0	0	-5	0	0	0	0	5	0	0	0
90	0	0	0	0	0	0	0	-5	0	0	0	0	0	5	0	0
91	0	0	0	0	0	0	0	-5	0	0	0	0	0	0	5	0
92	0	0	0	0	0	0	0	-5	0	0	0	0	0	0	0	5
93	0	0	0	0	0	0	0	0	-5	5	0	0	0	0	0	0
94	0	0	0	0	0	0	0	0	-5	0	5	0	0	0	0	0

95	0	0	0	0	0	0	0	0	-5	0	0	5	0	0	0	0
96	0	0	0	0	0	0	0	0	-5	0	0	0	5	0	0	0
97	0	0	0	0	0	0	0	0	-5	0	0	0	0	5	0	0
98	0	0	0	0	0	0	0	0	-5	0	0	0	0	0	5	0
99	0	0	0	0	0	0	0	0	-5	0	0	0	0	0	0	5
100	0	0	0	0	0	0	0	0	0	-5	5	0	0	0	0	0
101	0	0	0	0	0	0	0	0	0	-5	0	5	0	0	0	0
102	0	0	0	0	0	0	0	0	0	-5	0	0	5	0	0	0
103	0	0	0	0	0	0	0	0	0	-5	0	0	0	5	0	0
104	0	0	0	0	0	0	0	0	0	-5	0	0	0	0	5	0
105	0	0	0	0	0	0	0	0	0	-5	0	0	0	0	0	5
106	0	0	0	0	0	0	0	0	0	0	-5	5	0	0	0	0
107	0	0	0	0	0	0	0	0	0	0	-5	0	5	0	0	0
108	0	0	0	0	0	0	0	0	0	0	-5	0	0	5	0	0
109	0	0	0	0	0	0	0	0	0	0	-5	0	0	0	5	0
110	0	0	0	0	0	0	0	0	0	0	-5	0	0	0	0	5
111	0	0	0	0	0	0	0	0	0	0	0	-5	5	0	0	0
112	0	0	0	0	0	0	0	0	0	0	0	-5	0	5	0	0
113	0	0	0	0	0	0	0	0	0	0	0	-5	0	0	5	0
114	0	0	0	0	0	0	0	0	0	0	0	-5	0	0	0	5
115	0	0	0	0	0	0	0	0	0	0	0	0	-5	5	0	0
116	0	0	0	0	0	0	0	0	0	0	0	0	-5	0	5	0
117	0	0	0	0	0	0	0	0	0	0	0	0	-5	0	0	5
118	0	0	0	0	0	0	0	0	0	0	0	0	0	-5	5	0
119	0	0	0	0	0	0	0	0	0	0	0	0	0	-5	0	5
120	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-5	5
121	5	-5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
122	5	0	-5	0	0	0	0	0	0	0	0	0	0	0	0	0
123	5	0	0	-5	0	0	0	0	0	0	0	0	0	0	0	0
124	5	0	0	0	-5	0	0	0	0	0	0	0	0	0	0	0
125	5	0	0	0	0	-5	0	0	0	0	0	0	0	0	0	0
126	5	0	0	0	0	0	-5	0	0	0	0	0	0	0	0	0
127	5	0	0	0	0	0	0	-5	0	0	0	0	0	0	0	0
128	5	0	0	0	0	0	0	0	-5	0	0	0	0	0	0	0
129	5	0	0	0	0	0	0	0	0	-5	0	0	0	0	0	0
130	5	0	0	0	0	0	0	0	0	0	-5	0	0	0	0	0
131	5	0	0	0	0	0	0	0	0	0	0	-5	0	0	0	0
132	5	0	0	0	0	0	0	0	0	0	0	0	-5	0	0	0
133	5	0	0	0	0	0	0	0	0	0	0	0	0	-5	0	0
134	5	0	0	0	0	0	0	0	0	0	0	0	0	0	-5	0
135	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-5
136	0	5	-5	0	0	0	0	0	0	0	0	0	0	0	0	0
137	0	5	0	-5	0	0	0	0	0	0	0	0	0	0	0	0
138	0	5	0	0	-5	0	0	0	0	0	0	0	0	0	0	0
139	0	5	0	0	0	-5	0	0	0	0	0	0	0	0	0	0
140	0	5	0	0	0	0	-5	0	0	0	0	0	0	0	0	0
141	0	5	0	0	0	0	0	-5	0	0	0	0	0	0	0	0
142	0	5	0	0	0	0	0	0	-5	0	0	0	0	0	0	0
143	0	5	0	0	0	0	0	0	0	-5	0	0	0	0	0	0
144	0	5	0	0	0	0	0	0	0	0	-5	0	0	0	0	0

145	0	5	0	0	0	0	0	0	0	0	0	-5	0	0	0	0
146	0	5	0	0	0	0	0	0	0	0	0	0	-5	0	0	0
147	0	5	0	0	0	0	0	0	0	0	0	0	0	-5	0	0
148	0	5	0	0	0	0	0	0	0	0	0	0	0	0	-5	0
149	0	5	0	0	0	0	0	0	0	0	0	0	0	0	0	-5
150	0	0	5	-5	0	0	0	0	0	0	0	0	0	0	0	0
151	0	0	5	0	-5	0	0	0	0	0	0	0	0	0	0	0
152	0	0	5	0	0	-5	0	0	0	0	0	0	0	0	0	0
153	0	0	5	0	0	0	-5	0	0	0	0	0	0	0	0	0
154	0	0	5	0	0	0	0	-5	0	0	0	0	0	0	0	0
155	0	0	5	0	0	0	0	0	-5	0	0	0	0	0	0	0
156	0	0	5	0	0	0	0	0	0	-5	0	0	0	0	0	0
157	0	0	5	0	0	0	0	0	0	0	-5	0	0	0	0	0
158	0	0	5	0	0	0	0	0	0	0	0	-5	0	0	0	0
159	0	0	5	0	0	0	0	0	0	0	0	0	-5	0	0	0
160	0	0	5	0	0	0	0	0	0	0	0	0	0	-5	0	0
161	0	0	5	0	0	0	0	0	0	0	0	0	0	0	-5	0
162	0	0	5	0	0	0	0	0	0	0	0	0	0	0	0	-5
163	0	0	0	5	-5	0	0	0	0	0	0	0	0	0	0	0
164	0	0	0	5	0	-5	0	0	0	0	0	0	0	0	0	0
165	0	0	0	5	0	0	-5	0	0	0	0	0	0	0	0	0
166	0	0	0	5	0	0	0	-5	0	0	0	0	0	0	0	0
167	0	0	0	5	0	0	0	0	-5	0	0	0	0	0	0	0
168	0	0	0	5	0	0	0	0	0	-5	0	0	0	0	0	0
169	0	0	0	5	0	0	0	0	0	0	-5	0	0	0	0	0
170	0	0	0	5	0	0	0	0	0	0	0	-5	0	0	0	0
171	0	0	0	5	0	0	0	0	0	0	0	0	-5	0	0	0
172	0	0	0	5	0	0	0	0	0	0	0	0	0	-5	0	0
173	0	0	0	5	0	0	0	0	0	0	0	0	0	0	-5	0
174	0	0	0	5	0	0	0	0	0	0	0	0	0	0	0	-5
175	0	0	0	0	5	-5	0	0	0	0	0	0	0	0	0	0
176	0	0	0	0	5	0	-5	0	0	0	0	0	0	0	0	0
177	0	0	0	0	5	0	0	-5	0	0	0	0	0	0	0	0
178	0	0	0	0	5	0	0	0	-5	0	0	0	0	0	0	0
179	0	0	0	0	5	0	0	0	0	-5	0	0	0	0	0	0
180	0	0	0	0	5	0	0	0	0	0	-5	0	0	0	0	0
181	0	0	0	0	5	0	0	0	0	0	0	-5	0	0	0	0
182	0	0	0	0	5	0	0	0	0	0	0	0	-5	0	0	0
183	0	0	0	0	5	0	0	0	0	0	0	0	0	-5	0	0
184	0	0	0	0	5	0	0	0	0	0	0	0	0	0	-5	0
185	0	0	0	0	5	0	0	0	0	0	0	0	0	0	0	-5
186	0	0	0	0	0	5	-5	0	0	0	0	0	0	0	0	0
187	0	0	0	0	0	5	0	-5	0	0	0	0	0	0	0	0
188	0	0	0	0	0	5	0	0	-5	0	0	0	0	0	0	0
189	0	0	0	0	0	5	0	0	0	-5	0	0	0	0	0	0
190	0	0	0	0	0	5	0	0	0	0	-5	0	0	0	0	0
191	0	0	0	0	0	5	0	0	0	0	0	-5	0	0	0	0
192	0	0	0	0	0	5	0	0	0	0	0	0	-5	0	0	0
193	0	0	0	0	0	5	0	0	0	0	0	0	0	-5	0	0
194	0	0	0	0	0	5	0	0	0	0	0	0	0	0	-5	0

195	0	0	0	0	0	5	0	0	0	0	0	0	0	0	0	-5
196	0	0	0	0	0	0	5	-5	0	0	0	0	0	0	0	0
197	0	0	0	0	0	0	5	0	-5	0	0	0	0	0	0	0
198	0	0	0	0	0	0	5	0	0	-5	0	0	0	0	0	0
199	0	0	0	0	0	0	5	0	0	0	-5	0	0	0	0	0
200	0	0	0	0	0	0	5	0	0	0	0	-5	0	0	0	0
201	0	0	0	0	0	0	5	0	0	0	0	0	-5	0	0	0
202	0	0	0	0	0	0	5	0	0	0	0	0	0	-5	0	0
203	0	0	0	0	0	0	5	0	0	0	0	0	0	0	-5	0
204	0	0	0	0	0	0	5	0	0	0	0	0	0	0	0	-5
205	0	0	0	0	0	0	0	5	-5	0	0	0	0	0	0	0
206	0	0	0	0	0	0	0	5	0	-5	0	0	0	0	0	0
207	0	0	0	0	0	0	0	5	0	0	-5	0	0	0	0	0
208	0	0	0	0	0	0	0	5	0	0	0	-5	0	0	0	0
209	0	0	0	0	0	0	0	5	0	0	0	0	-5	0	0	0
210	0	0	0	0	0	0	0	5	0	0	0	0	0	-5	0	0
211	0	0	0	0	0	0	0	5	0	0	0	0	0	0	-5	0
212	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0	-5
213	0	0	0	0	0	0	0	0	5	-5	0	0	0	0	0	0
214	0	0	0	0	0	0	0	0	5	0	-5	0	0	0	0	0
215	0	0	0	0	0	0	0	0	5	0	0	-5	0	0	0	0
216	0	0	0	0	0	0	0	0	5	0	0	0	-5	0	0	0
217	0	0	0	0	0	0	0	0	5	0	0	0	0	-5	0	0
218	0	0	0	0	0	0	0	0	5	0	0	0	0	0	-5	0
219	0	0	0	0	0	0	0	0	5	0	0	0	0	0	0	-5
220	0	0	0	0	0	0	0	0	0	5	-5	0	0	0	0	0
221	0	0	0	0	0	0	0	0	0	5	0	-5	0	0	0	0
222	0	0	0	0	0	0	0	0	0	5	0	0	-5	0	0	0
223	0	0	0	0	0	0	0	0	0	5	0	0	0	-5	0	0
224	0	0	0	0	0	0	0	0	0	5	0	0	0	0	-5	0
225	0	0	0	0	0	0	0	0	0	5	0	0	0	0	0	-5
226	0	0	0	0	0	0	0	0	0	0	5	-5	0	0	0	0
227	0	0	0	0	0	0	0	0	0	0	5	0	-5	0	0	0
228	0	0	0	0	0	0	0	0	0	0	5	0	0	-5	0	0
229	0	0	0	0	0	0	0	0	0	0	5	0	0	0	-5	0
230	0	0	0	0	0	0	0	0	0	0	5	0	0	0	0	-5
231	0	0	0	0	0	0	0	0	0	0	0	5	-5	0	0	0
232	0	0	0	0	0	0	0	0	0	0	0	5	0	-5	0	0
233	0	0	0	0	0	0	0	0	0	0	0	5	0	0	-5	0
234	0	0	0	0	0	0	0	0	0	0	0	5	0	0	0	-5
235	0	0	0	0	0	0	0	0	0	0	0	0	5	-5	0	0
236	0	0	0	0	0	0	0	0	0	0	0	0	5	0	-5	0
237	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0	-5
238	0	0	0	0	0	0	0	0	0	0	0	0	0	5	-5	0
239	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	-5
240	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	-5
241	5	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0
242	0	5	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0
243	0	0	5	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	0
244	0	0	0	5	-1	-1	-1	-1	-1	0	0	0	0	0	0	0

245	0	0	0	0	5	-1	-1	-1	-1	-1	0	0	0	0	0	0
246	0	0	0	0	0	5	-1	-1	-1	-1	-1	0	0	0	0	0
247	0	0	0	0	0	0	5	-1	-1	-1	-1	-1	0	0	0	0
248	0	0	0	0	0	0	0	5	-1	-1	-1	-1	-1	0	0	0
249	0	0	0	0	0	0	0	0	5	-1	-1	-1	-1	-1	0	0
250	0	0	0	0	0	0	0	0	0	5	-1	-1	-1	-1	-1	0
251	0	0	0	0	0	0	0	0	0	0	5	-1	-1	-1	-1	-1
252	-1	0	0	0	0	0	0	0	0	0	0	5	-1	-1	-1	-1
253	-1	-1	0	0	0	0	0	0	0	0	0	0	5	-1	-1	-1
254	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	5	-1	-1
255	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	5	-1
256	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	5
257	-5	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
258	0	-5	1	1	1	1	1	0	0	0	0	0	0	0	0	0
259	0	0	-5	1	1	1	1	1	0	0	0	0	0	0	0	0
260	0	0	0	-5	1	1	1	1	1	0	0	0	0	0	0	0
261	0	0	0	0	-5	1	1	1	1	1	0	0	0	0	0	0
262	0	0	0	0	0	-5	1	1	1	1	1	0	0	0	0	0
263	0	0	0	0	0	0	-5	1	1	1	1	1	0	0	0	0
264	0	0	0	0	0	0	0	-5	1	1	1	1	1	0	0	0
265	0	0	0	0	0	0	0	0	-5	1	1	1	1	1	0	0
266	0	0	0	0	0	0	0	0	0	-5	1	1	1	1	1	0
267	0	0	0	0	0	0	0	0	0	0	-5	1	1	1	1	1
268	1	0	0	0	0	0	0	0	0	0	0	-5	1	1	1	1
269	1	1	0	0	0	0	0	0	0	0	0	0	-5	1	1	1
270	1	1	1	0	0	0	0	0	0	0	0	0	0	-5	1	1
271	1	1	1	1	0	0	0	0	0	0	0	0	0	0	-5	1
272	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	-5
273	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
274	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
275	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
276	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
277	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
278	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
279	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
280	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
281	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
282	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
283	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
284	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

APPENDIX Q: SSN CCD Design Matrix.

Run	X12	X17	X19	X21	X24	X30	X33	X40	X41	X43	X53	X60	X64
1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	-1	1	1	-1	-1	-1
3	1	1	1	1	1	1	-1	1	1	1	-1	1	1
4	1	1	1	1	1	1	-1	-1	1	1	1	-1	-1
5	1	1	1	1	1	-1	1	1	1	-1	-1	-1	-1
6	1	1	1	1	1	-1	1	-1	1	-1	1	1	1
7	1	1	1	1	1	-1	-1	1	1	-1	1	-1	-1
8	1	1	1	1	1	-1	-1	-1	1	-1	-1	1	1
9	1	1	1	1	-1	1	1	1	1	-1	-1	1	1
10	1	1	1	1	-1	1	1	-1	1	-1	1	-1	-1
11	1	1	1	1	-1	1	-1	1	1	-1	1	1	1
12	1	1	1	1	-1	1	-1	-1	1	-1	-1	-1	-1
13	1	1	1	1	-1	-1	1	1	1	1	1	-1	-1
14	1	1	1	1	-1	-1	1	-1	1	1	-1	1	1
15	1	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1
16	1	1	1	1	-1	-1	-1	-1	1	1	1	1	1
17	1	1	1	-1	1	1	1	1	-1	1	1	-1	1
18	1	1	1	-1	1	1	1	-1	-1	1	-1	1	-1
19	1	1	1	-1	1	1	-1	1	-1	1	-1	-1	1
20	1	1	1	-1	1	1	-1	-1	-1	1	1	1	-1
21	1	1	1	-1	1	-1	1	1	-1	-1	-1	1	-1
22	1	1	1	-1	1	-1	1	-1	-1	-1	1	-1	1
23	1	1	1	-1	1	-1	-1	1	-1	-1	1	1	-1
24	1	1	1	-1	1	-1	-1	-1	-1	-1	-1	-1	1
25	1	1	1	-1	-1	1	1	1	-1	-1	-1	-1	1
26	1	1	1	-1	-1	1	1	-1	-1	-1	1	1	-1
27	1	1	1	-1	-1	1	-1	1	-1	-1	1	-1	1
28	1	1	1	-1	-1	1	-1	-1	-1	-1	-1	1	-1
29	1	1	1	-1	-1	-1	1	1	-1	1	1	1	-1
30	1	1	1	-1	-1	-1	1	-1	-1	1	-1	-1	1
31	1	1	1	-1	-1	-1	-1	1	-1	1	-1	1	-1
32	1	1	1	-1	-1	-1	-1	-1	-1	1	1	-1	1
33	1	1	-1	1	1	1	1	1	-1	1	1	1	1
34	1	1	-1	1	1	1	1	-1	-1	1	-1	-1	-1
35	1	1	-1	1	1	1	-1	1	-1	1	-1	1	1
36	1	1	-1	1	1	1	-1	-1	-1	1	1	-1	-1
37	1	1	-1	1	1	-1	1	1	-1	-1	-1	-1	-1
38	1	1	-1	1	1	-1	1	-1	-1	-1	1	1	1
39	1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	-1
40	1	1	-1	1	1	-1	-1	-1	-1	-1	-1	1	1
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268	0	0	0	0	0	-1	0	0	0	0	0	0	0
269	0	0	0	0	0	0	1	0	0	0	0	0	0
270	0	0	0	0	0	0	-1	0	0	0	0	0	0
271	0	0	0	0	0	0	0	1	0	0	0	0	0
272	0	0	0	0	0	0	0	-1	0	0	0	0	0
273	0	0	0	0	0	0	0	0	1	0	0	0	0
274	0	0	0	0	0	0	0	0	-1	0	0	0	0
275	0	0	0	0	0	0	0	0	0	1	0	0	0
276	0	0	0	0	0	0	0	0	0	-1	0	0	0
277	0	0	0	0	0	0	0	0	0	0	1	0	0
278	0	0	0	0	0	0	0	0	0	0	-1	0	0
279	0	0	0	0	0	0	0	0	0	0	0	1	0
280	0	0	0	0	0	0	0	0	0	0	0	-1	0
281	0	0	0	0	0	0	0	0	0	0	0	0	1
282	0	0	0	0	0	0	0	0	0	0	0	0	-1
283	0	0	0	0	0	0	0	0	0	0	0	0	0
284	0	0	0	0	0	0	0	0	0	0	0	0	0
285	0	0	0	0	0	0	0	0	0	0	0	0	0
286	0	0	0	0	0	0	0	0	0	0	0	0	0
287	0	0	0	0	0	0	0	0	0	0	0	0	0
288	0	0	0	0	0	0	0	0	0	0	0	0	0
289	0	0	0	0	0	0	0	0	0	0	0	0	0
290	0	0	0	0	0	0	0	0	0	0	0	0	0
291	0	0	0	0	0	0	0	0	0	0	0	0	0
292	0	0	0	0	0	0	0	0	0	0	0	0	0
293	0	0	0	0	0	0	0	0	0	0	0	0	0
294	0	0	0	0	0	0	0	0	0	0	0	0	0
295	0	0	0	0	0	0	0	0	0	0	0	0	0
296	0	0	0	0	0	0	0	0	0	0	0	0	0

297	0	0	0	0	0	0	0	0	0	0	0	0	0
298	0	0	0	0	0	0	0	0	0	0	0	0	0
299	0	0	0	0	0	0	0	0	0	0	0	0	0
300	0	0	0	0	0	0	0	0	0	0	0	0	0

APPENDIX R: 20-Term Design Centers.

Variable	Optimal	Gradient	Quadratic
X1	79	78	77
X2	25	25	25
X3	55	55	55
X4	14	14	15
X5	0	0	0
X6	0	0	0
X7	26	26	26
X8	0	0	0
X9	0	0	0
X10	27	27	27
X11	34	35	35
X12	40	40	40
X13	40	40	40
X14	31	31	31
X15	37	37	37
X16	30	30	30
X17	46	46	46
X18	33	33	33
X19	48	48	48
X20	0	0	0
X21	35	35	35
X22	130	129	104
X23	13	13	16
X24	29	29	32
X25	6	6	9
X26	0	0	0
X27	0	0	0
X28	14	14	14
X29	0	0	0
X30	0	0	0
X31	14	14	17
X32	18	18	21
X33	20	20	23
X34	20	21	22
X35	15	15	15
X36	19	19	22
X37	17	17	17
X38	25	25	25
X39	17	17	20
X40	25	25	25
X41	0	0	0
X42	18	18	18
X43	383	386	386
X44	0	0	0
X45	0	0	0

X46	0	0	0
X47	35	35	35
X48	43	43	43
X49	0	0	0
X50	19	19	19
X51	17	17	17
X52	0	0	0
X53	0	0	0
X54	0	0	0
X55	0	0	0
X56	0	0	0
X57	0	0	0
X58	0	0	0
X59	0	0	0
X60	0	0	0
X61	0	0	0
X62	29	30	29
X63	0	0	0

APPENDIX S, SSN Design Center.

Variable	Optimal	Variable	Optimal
X1	0	X46	0
X2	0	X47	5
X3	31	X48	10
X4	0	X49	0
X5	18	X50	0
X6	3	X51	2
X7	0	X52	22
X8	0	X53	9
X9	9	X54	26
X10	0	X55	13
X11	11	X56	4
X12	21	X57	0
X13	69	X58	4
X14	0	X59	2
X15	0	X60	7
X16	44	X61	0
X17	10	X62	0
X18	0	X63	11
X19	18	X64	24
X20	3	X65	0
X21	10	X66	127
X22	5	X67	0
X23	22	X68	0
X24	18	X69	0
X25	0	X70	0
X26	0	X71	0
X27	4	X72	0
X28	24	X73	0
X29	17	X74	0
X30	36	X75	3
X31	26	X76	1
X32	0	X77	2
X33	22	X78	3
X34	83	X79	3
X35	0	X80	0
X36	77	X81	2
X37	8	X82	2
X38	0	X83	0
X39	14	X84	0
X40	8	X85	3
X41	18	X86	1
X42	0	X87	1
X43	39	X88	3
X44	42	X89	0
X45	5		

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Vita

Lt. David Mills was born in San Antonio, Texas on August 22, 1974, the 4th child of Jess Mills III and Linda Mills. He grew up in Pleasanton, Texas, received a General Equivalency Diploma on May 21, 1992, and attended Palo Alto College in San Antonio and Southwest Texas State University in San Marcos, TX. He graduated *Magna Cum Laude* from Southwest Texas State with a B.S. in Mathematics in May 1997, and was commissioned through the ROTC detachment on May 9, 1997. His first assignment was to the Air Force Institute of Technology in Dayton, Ohio, where he pursued an M.S. in Operations Research. Upon graduation he was assigned to the National Security Agency in Columbia, Maryland.

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Research Impact Statement

Student	Faculty Advisor	Thesis Designator	Keyword #1	Keyword #2
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Related Thesis #1	Related Thesis #2	Related Thesis #3	Related Thesis #4

Title: Two-Stage Stochastic Linear Programming with Recourse: A Characterization of Local Regions using Response Surface Methodology

Subject: Characterization of the surface of the two-stage stochastic linear program with recourse family of problems for sensitivity analysis

Air Force Program Description:

The multiple-stage linear programming with recourse family of problems has been used by the USAF in the past to model mobility issues. A great deal of work has been done regarding the optimization of such models, but little work has been accomplished regarding the sensitivity of the solution to change.

Impact Statement:

This work provides a methodology for characterizing the surface of the two-stage stochastic linear programming with recourse family of problems. One of the key advantages of this type of analysis is the sensitivity insight gained from the final fitted model. This sensitivity analysis can assist a decision maker faced with issues not originally included in the model.

Technical Abstract:

The LP recourse problem applies to two-stage optimization problems where uncertainty in resource availability of the second stage hinders informed decision making. The recourse function affords a way to compensate "later" for an error in prediction "now." The literature provides a rich body of work on the optimization of such problems, but little research has been accomplished regarding the characterization of the surface in the local region of optimality, in particular sensitivity analysis. A decision maker faced with considerations other than the modeled objective function must be presented with a way to estimate the impact of operating at non-optimal decision variable values. This work develops and demonstrates a technique for characterizing the surface using response surface methodology. Specifically, the flexibility and utility of RSM techniques applied to this class of problems is demonstrated, and a methodology for characterizing the surface in the local region using a low-order polynomial develop is developed.

Subject Terms: Stochastic Linear Programming, Response Surface Methodology

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The LP recourse problem applies to two-stage optimization problems where uncertainty in resource availability of the second stage hinders informed decision making. The recourse function affords a way to compensate "later" for an error in prediction "now." The literature provides a rich body of work on the optimization of such problems, but little research has been accomplished regarding the characterization of the surface in the local region of optimality, in particular sensitivity analysis. A decision maker faced with considerations other than the modeled objective function must be presented with a way to estimate the impact of operating at non-optimal decision variable values. This work develops and demonstrates a technique for characterizing the surface using response surface methodology. Specifically, the flexibility and utility of RSM techniques applied to this class of problems is demonstrated, and a methodology for characterizing the surface in the local region using a low-order polynomial is developed.

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